What is a random number and why should I care?

Frances Buontempo

A random number

```
int GetRandom()
{
    return 59;
    //A perfectly randomly
    // picked number
}
```

http://stackoverflow.com/questions/4195958/how-do-i-scale-down-numbers-from-rand

What is random?

- A single number is not random, a sequence of numbers might be (Knuth)
- They still have properties
  - Mean (expectations)
  - Variance
- Chaotic != random
- Pseudo-random numbers

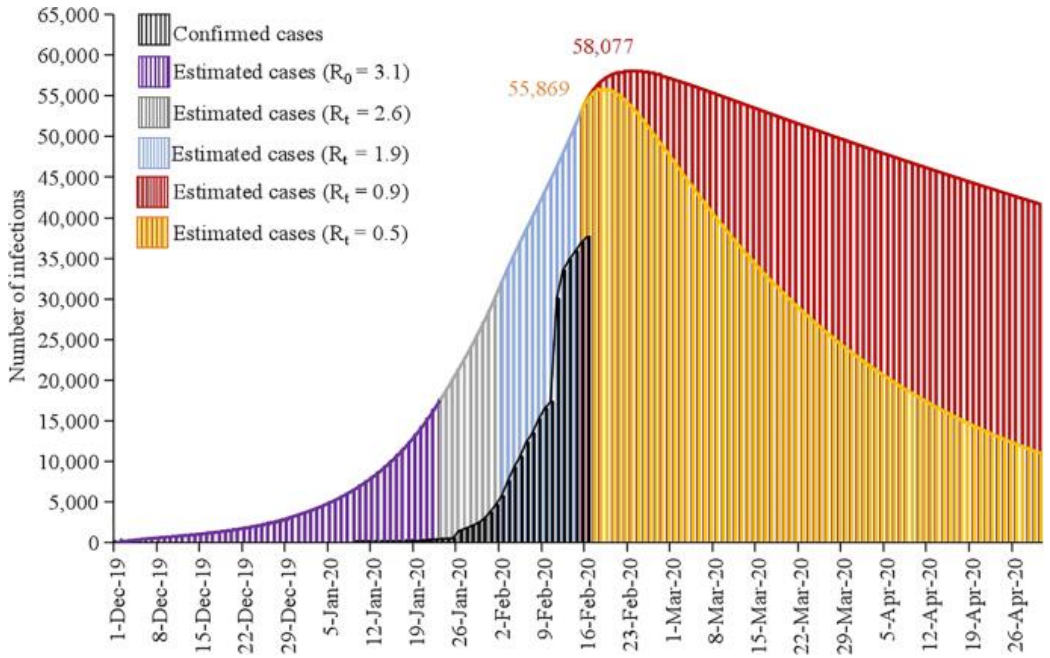
It is impossible to prove definitively whether a given sequence of numbers is random.

"Anyone who attempts to generate random numbers by deterministic means is, of course, living in a state of sin."

John von Neumann

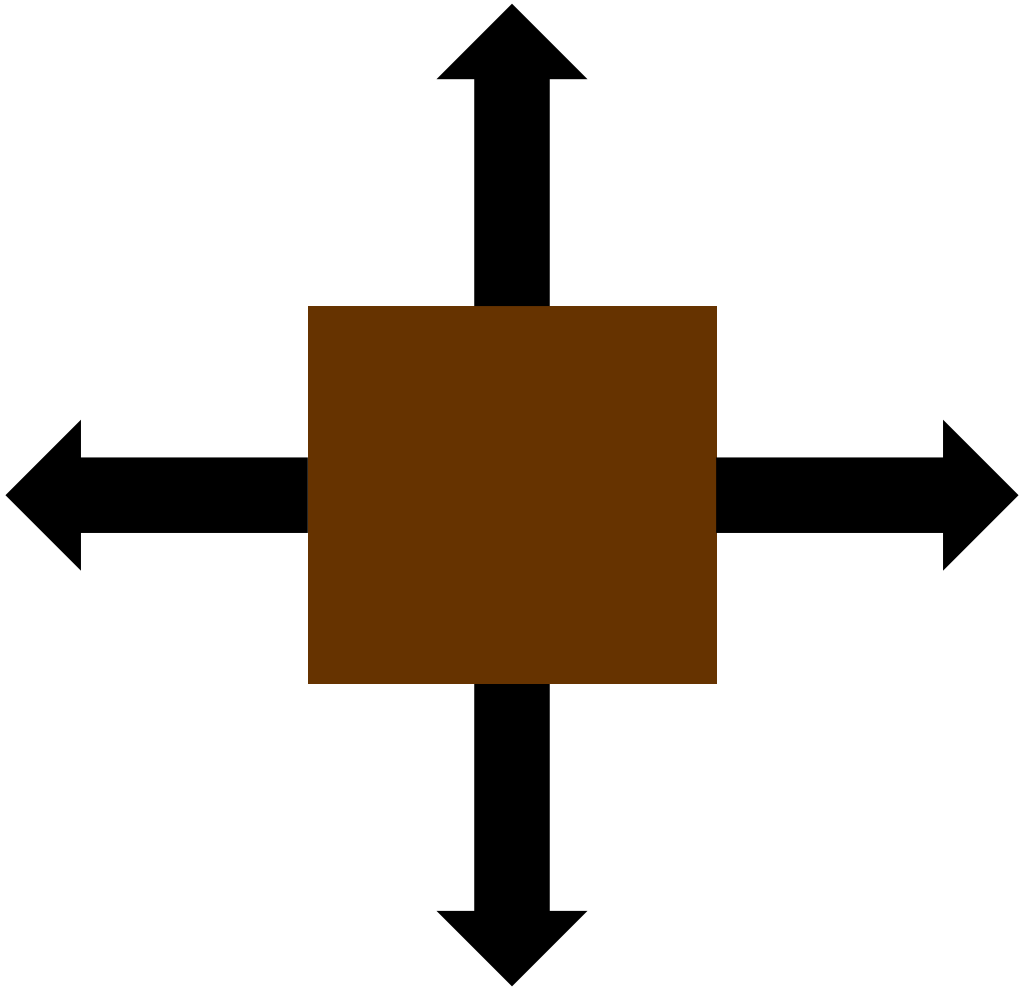
Why should you care?

- People get it wrong
- We need randomness for
  - Games
  - Simulations, e.g. Covid-19 modeling
  - …
- How to test
  - Code using random numbers
  - Random number generators

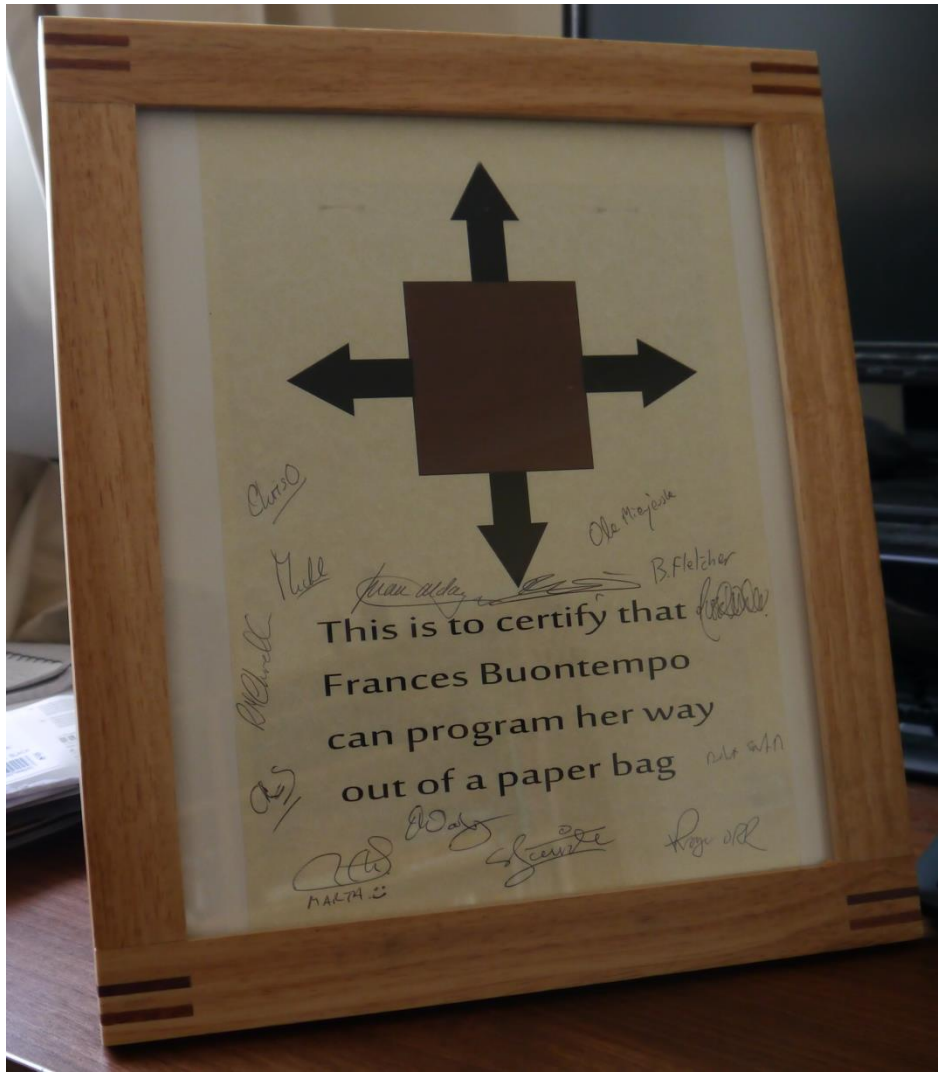


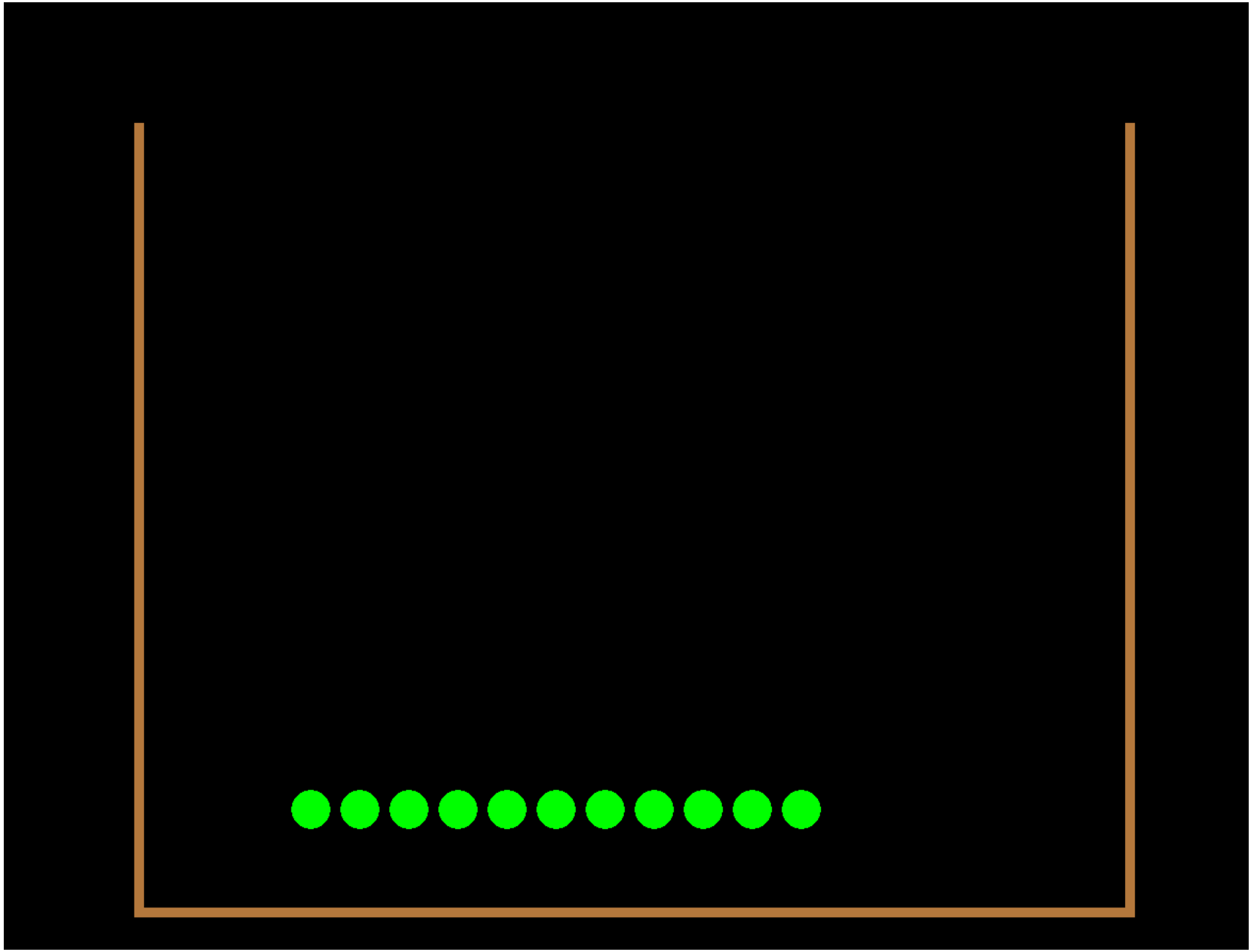
https://www.nature.com/articles/s41421-020-0148-0
infectious disease dynamics SEIR (Susceptible, Exposed, Infectious, and Removed) model

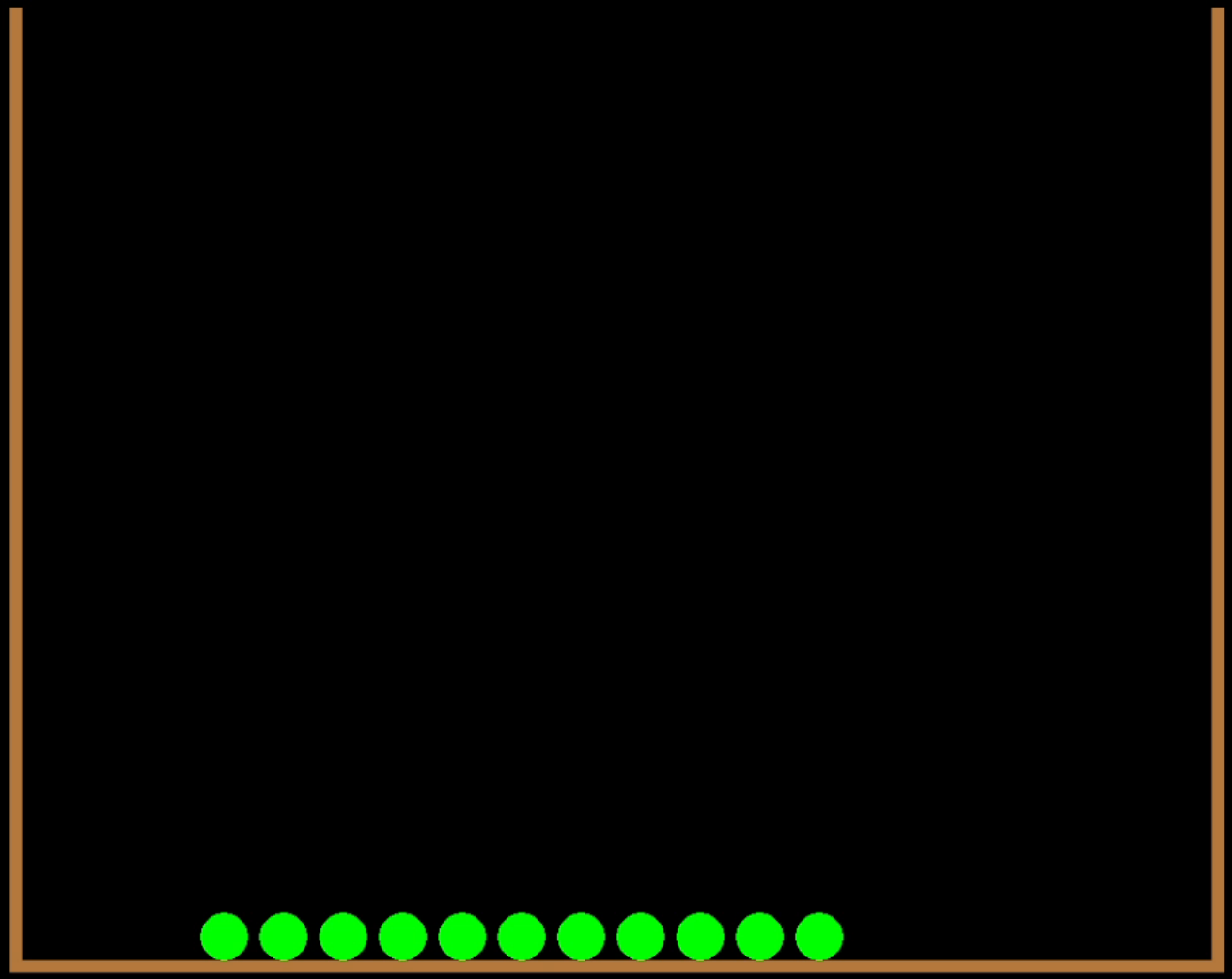
Code your way out of a paper bag!



Award!


This is to certify that
Frances Buontempo
can program her way
out of a paper bag





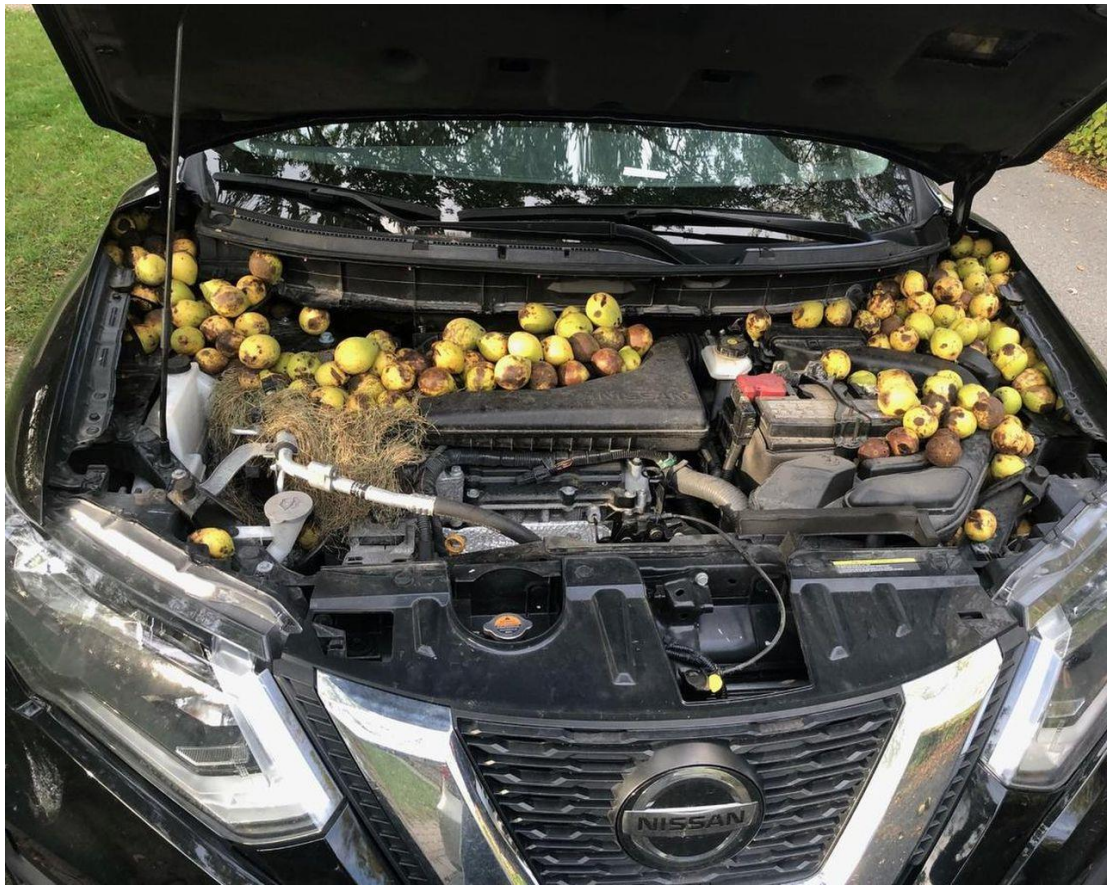
How do you generate random outcomes?

- C’s rand
- Python’s random.random()
- C#’s Random.Next(1, 7)
- C#’s System.Security.Cryptography.RandomNumberGenerator
- rlang runif(1), rnorm(4)

Always ask two questions

1. Default seed?
2. Range (), [) or []?

Under the hood



From https://www.thestar.com/opinion/star-columnists/2021/10/25/this-is-nuts-after-squirrels-hid-hundreds-of-walnuts-under-the-hood-of-my-car-i-went-in-search-for-answers.html

Generating random numbers

- Linear congruential generator (1958 by W. E. Thomson and A. Rotenber)
- $x_{i+1} = (Ax_i + c) mod\ M$
- If c is zero, it's a multiplicative linear congruential generator (MLCG) or multiplicative congruential generator (MCG).
  - Or a Lehmer RNG (1951)
  - $x_{i+1} = Ax_i mod\ M$
  - With e.g. M = $2^{31} - 1$ = 2,147,483,647 (a Mersenne prime) and A is 7^5 = 16,807

Cycle or Collapse

- $x_{i+1} = A\ x_i \bmod M$
- Up to M-1 if M is prime and we choose a suitable A
  - A=7 and M=11: **1,** 7, 5, 2, 3, 10, 4, 6, 9, 8, **1**, …
  - A=5 and M=11: **1,** 5, 3, 4, 9, **1**, …
- Why prime (or coprime)?
  - A=4 and M=12 starting with 1: 1, 4, 4, 4, …
    - 4*1mod 12 = 4, then 4*4 mod 12 = 16 mod 12 = 4, …
  - A=24 and M=12 starting with 1: 1, 0, 0, 0, …
    - 24*1 mod 12 = 0, …

Mersenne Twister

- $x_{k+n} = x_{k+m} \oplus \left(\left(x_k^u | x_{k+1}^l \right) A \right) \; k = 0, 1, \ldots$
  - $|$ concatenation of bit vectors
  - $\oplus$ bitwise exclusive or
  - A is the "twist transformation"
  - $xA = \begin{cases} x \gg 1, lowest\ bit\ of\ x, x_0 = 0 \\ (x \gg 1) \oplus a, x_0 = 1 \end{cases}$
- Several seeds, to make bits for x=$(x_{w-1}, x_{w-2}, \ldots, x_0,)$
- Matsumoto and Nishimura ACM Transactions on Modeling and Computer Simulation Vol 8 Issue 1 Jan. 1998 pp 3–30
  - https://dl.acm.org/doi/10.1145/272991.272995

Other engines are available

- Subtract with carry
  - AKA lagged Fibonacci: new term is “some combination” of any 2 previous terms
- Engine adaptors: generate pseudo-random numbers using another random number engine as entropy source.
  - discard_block_engine
    - discards some output of a random number engine
  - independent_bits_engine
    - packs the output of a random number engine into blocks of a specified number of bits
  - shuffle_order_engine
    - delivers the output of a random number engine in a different order

Is random a W.I.P?

- minstd_rand0
  - std::linear_congruential_engine<std::uint_fast32_t, **16807**, 0, 2147483647>
  - Discovered in 1969 by Lewis, Goodman and Miller, adopted as "Minimal standard" in 1988 by Park and Miller
- minstd::rand
  - std::linear_congruential_engine<std::uint_fast32_t, **48271**, 0, 2147483647>
  - Newer "Minimum standard", recommended by Park, Miller, and Stockmeyer in 1993
- Proposal P1932
  - The XorShift and Philox class generators are good candidates.

https://wg21.link/P1932

- Python's numpy default_rng
  - Now using PCG64 = permuted congruential generator, from 2014

https://numpy.org/doc/stable/reference/random/index.html#random-quick-start

P1932

Each of the C++11 random number generators has own advantages and disadvantages in terms of described criteria,

e.g. linear congruential generators, the simplest generators with 32-bit state, have a quite short generation period (2^32) and weak statistical properties

while Mersenne Twister 19937 generator has long generation period and strong statistical properties relying a big vector state underneath; in its turn, this state impacts on the effective support of parallel Monte Carlo simulations.

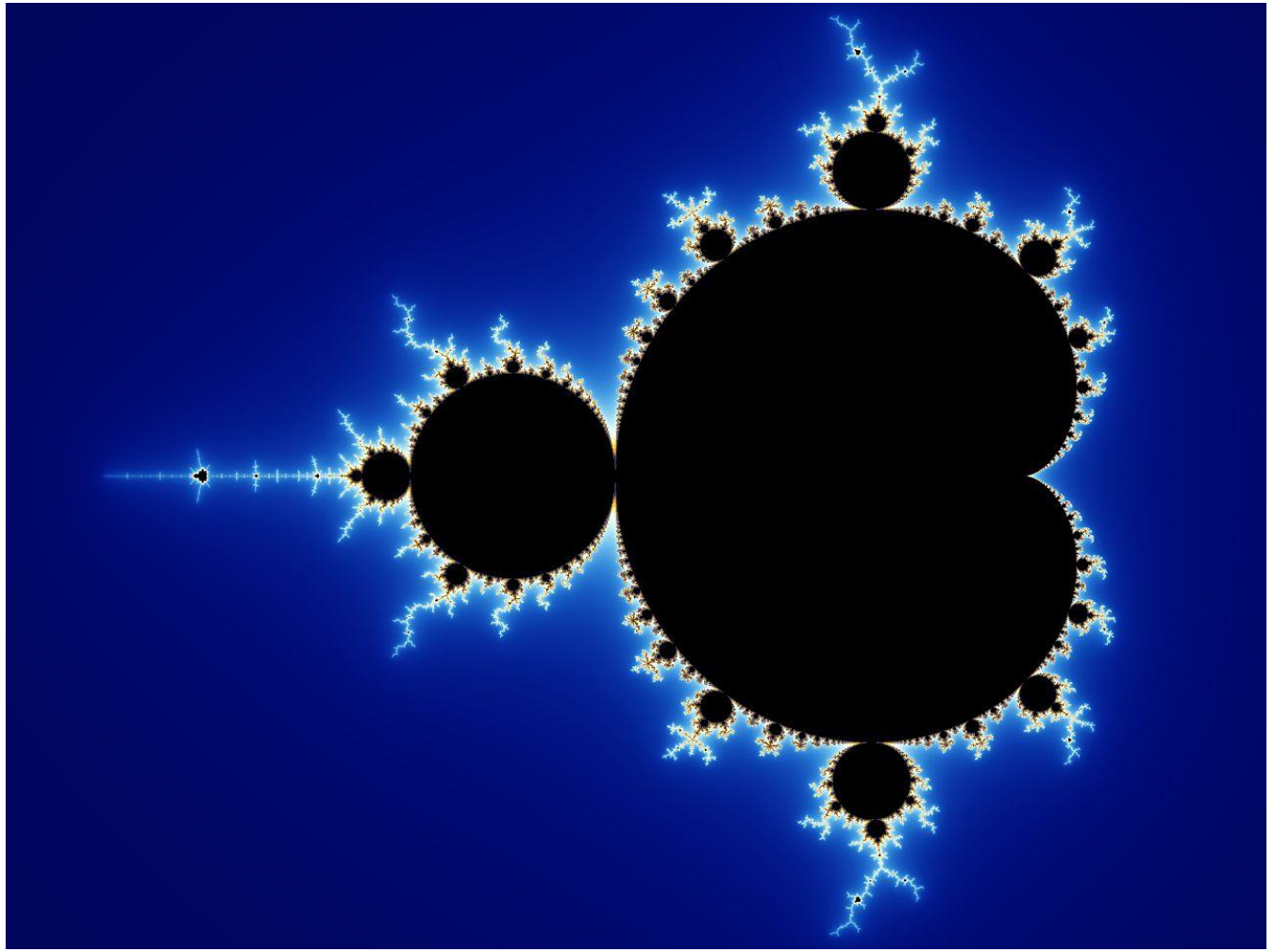
C++ random number generators do not support additional use cases such as quasi Monte Carlo simulations.

(An aside) $x^2 - 1$

1	0	0.25	0.5
0	-1	-0.9375	-0.75
-1	0	-0.12109	-0.4375
0	-1	-0.98534	-0.80859
-1	0	-0.02911	-0.34618
0	-1	-0.99915	-0.88016
-1	0	-0.00169	-0.22531
0	-1	-1	-0.94923
-1	0	-5.7E-06	-0.09896
0	-1	-1	-0.99021
-1	0	-6.6E-11	-0.01949
0	-1	-1	-0.99962
-1	0	0	-0.00076
0	-1	-1	-1
-1	0	0	-1.2E-06
0	-1	-1	-1
-1	0	0	-2.7E-12
0	-1	-1	-1
-1	0	0	0

$$z_{n+1} = z_n^2 - c$$

- Start with z=0, and pick a complex number c.
- Cycle, collapse (black), or continue growing (colour)…



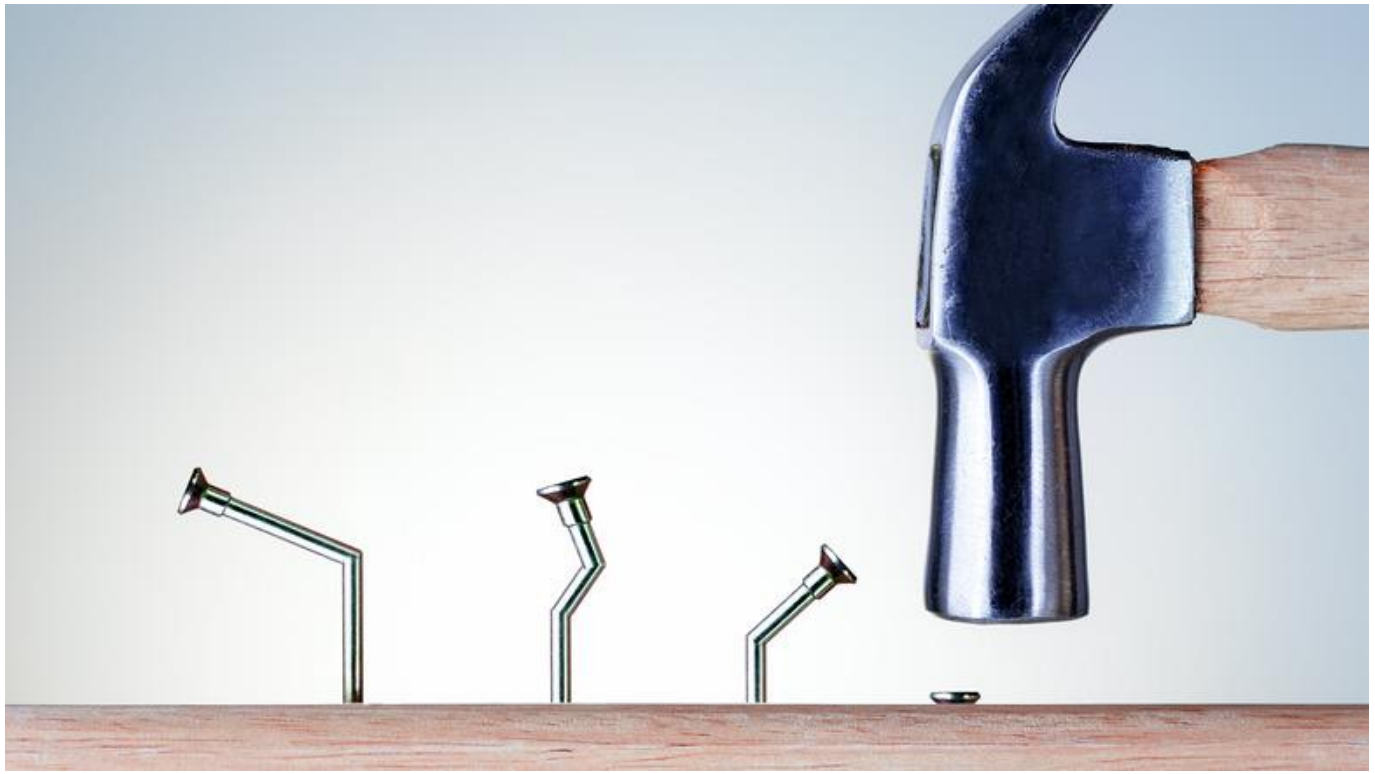
Created by Wolfgang Beyer with the program Ultra Fractal 3. - Own work, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=321973

Or $2x^2 - 1$?

1	0	0.5	0.25
1	-1	-0.5	-0.875
1	1	-0.5	0.53125
1	1	-0.5	-0.43555
1	1	-0.5	-0.6206
1	1	-0.5	-0.22972
1	1	-0.5	-0.89446
1	1	-0.5	0.600119
1	1	-0.5	-0.27971
1	1	-0.5	-0.84352
1	1	-0.5	0.423054
1	1	-0.5	-0.64205
1	1	-0.5	-0.17554
1	1	-0.5	-0.93837
1	1	-0.5	0.761082
1	1	-0.5	0.158491
1	1	-0.5	-0.94976
1	1	-0.5	0.804093
1	1	-0.5	0.293132

Using random number generators

- Time to think
- Possible problems
- Common use cases
- Testing….



Seeds

- The number the recurrence relationship starts with x_i
- What should you use?
  - Current time?
  - Known number?
- **Same seed, different languages… (or same lang different os, etc)**

Threads

- Might not be thread safe… global state
  - “It is implementation-defined whether rand() is thread-safe.”
  - https://en.cppreference.com/w/cpp/numeric/random/rand
- Two threads… different results depending on order
  - Not just in C
- Don’t use C’s rand
  - https://learn.microsoft.com/en-us/events/goingnative-2013/rand-considered-harmful

Roll a die in Python

```
import random
random.randint(1,6)#alias for randrange(1, 6+1)
```

Always ask two questions

1. Default seed?
2. Range (), [) or []?

Never use %

- In C….

```
srand(time(null));
```

- https://c-faq.com/lib/randrange.html

```
rand() % N          /* POOR */
```

rand returns RAND_MAX+1 distinct values, which cannot always be evenly divvied up into N buckets

```
M + rand() / (RAND_MAX / (N - M + 1) + 1)
```

gives numbers in the range [M, N]

Roll a die in C++

```
#include <iostream>
#include <random>
int main()
{
    std::default_random_engine generator; // probably a mt19937
    std::uniform_int_distribution<int> distribution(1, 6);
    const int count = 3;
    for (int i = 0; i < count; ++i)
    {
        std::cout << distribution(generator) << '\n';
    }
}
```

```
using default_random_engine = mt19937;

using mt19937 = mersenne_twister_engine<unsigned int, 32, 624, 397, 31,
0x9908b0df, 11, 0xffffffff, 7, 0x9d2c5680, 15, 0xefc60000, 18, 1812433253>;

class mersenne_twister_engine : public mersenne_twister<_Ty, _Wx, _Nx,
_Mx, _Rx, _Px, _Ux, _Sx, _Bx, _Tx, _Cx, _Lx>;

class mersenne_twister : public _Circ_buf<_Ty, _Nx> {
    static constexpr _Ty default_seed = 5489U;
    mersenne_twister() : _Dxval(_WMSK) {
        seed(default_seed, static_cast<_Ty>(1812433253));
    }
}
```

Two questions

```
#include <iostream>
#include <random>
int main()
{
    std::random_device rd;
    std::default_random_engine generator{rd()};
    std::uniform_int_distribution distribution(1, 6); // don’t need <int>
    const int count = 3;
    for (int i = 0; i < count; ++i)
    {
        std::cout << distribution(generator) << '\n';
    }
}
```

Could use time… but

```
#include <chrono>


std::default_random_engine generator{
static_cast<unsigned int>(
    std::chrono::steady_clock::now().
        time_since_epoch().count())
};
```

A ~~warning~~ discussion about seeds

The current standard library does not provide any convenient way to use a std::random_device to properly (in the sense that each initial state is equally likely) seed a random engine.

The naïve approach that most people seem to use is the following.

```
template <typename EngineT>   //
requires(RandomNumberEngine(EngineT))
void seed_non_deterministically_1st(EngineT& engine)
{
  std::random_device rnddev {};
  engine.seed(rnddev());
}
```

This code is severely flawed. If EngineT is std::mt19937, it has a state size of 19968 bits. However, if an unsigned int is 32 bits (as is the common case on many platforms today), then of the up to 2^{19968} states, at most 2^{32} (that is one 2^{-19936}-th) can possibly be chosen!

https://www.open-std.org/jtc1/sc22/wg21/docs/papers/2016/p0205r0.html

More weirdness

- The Mersenne Twister uses 624 32-bit integers to represent its internal state, plus a few more for housekeeping, so using one 32 bit value (e.g. from random device) seems odd.
- “Strangely enough, when you initialize the Mersenne Twister with a 32-bit seed (via seed_seq), **it can't ever generate 7, or 13 as its first output**. And **two different seeds produce 0**. Even more crazy, there are **twelve different 32-bit seeds that can produce the "random" numbers 1226181350 and 1563636090**, so those numbers show up twelve times more often than we'd expect.”

https://www.pcg-random.org/posts/cpp-seeding-surprises.html

Oh really?

```
std::seed_seq seeder{ 1080100664 };
std::mt19937 rng(seeder);
std::cout << rng() << '\n';
```

versus

```
std::mt19937 rng(1080100664);
std::cout << rng() << '\n';
```

Warm up?

```
auto RandomlySeededMersenneTwister () {
    std::mt19937 rng(std::random_device{}());
    rng.discard(700000);
    return rng;
}
```

https://codereview.stackexchange.com/questions/109260/seed-stdmt19937-from-stdrandom-device

700000 from "Improved long-period generators based on linear recurrences modulo 2", F. Panneton, P. L'Ecuyer, M. Matsumoto in ACM TOMS Volume 32 Issue 1, March 2006 Pages 1-16

But

"seed_seq initialization used by std::mt19937 performs a warm up"

https://www.learncpp.com/cpp-tutorial/generating-random-numbers-using-mersenne-twister/

```
mersenne_twister_engine() : _Mybase(default_seed, _Dx, _Fx) {}

explicit mersenne_twister_engine(result_type _Xx0) :
     _Mybase(_Xx0, _Dx, _Fx) {}

template <class _Seed_seq, _Enable_if_seed_seq_t<_Seed_seq,
mersenne_twister_engine> = 0>
explicit mersenne_twister_engine(_Seed_seq& _Seq) :
_Mybase(default_seed, _Dx, _Fx) {
        seed(_Seq);
}
```

My head hurts!

- First, C++ standard randoms are not portable ("seed-stable")
  - But see https://www.open-std.org/jtc1/sc22/wg21/docs/papers/2020/p2059r0.pdf
- 2nd this is good enough:

```
std::random_device rd;
std::mt19937 rng{ rd() };
```

- 3rd this might be slightly better:

```
std::random_device rd;
std::seed_seq seeder{ rd() };
std::mt19937 rng(seeder);
```

Roll two dice

Can they share the same generator?

```
rng1 = random.Random()
rng2 = random.Random()
rng1.randint(1,6)
rng2.randint(1,6)
```

Roll two dice in C++

```
std::random_device rd;
std::default_random_engine generator1{ rd() };
std::default_random_engine generator2{ rd() };
  //Using time instead of rd would be bad!
std::uniform_int_distribution distribution(1, 6);
const int count = 3;
for (int i = 0; i < count; ++i)
{
    std::cout << distribution(generator1) << '\n';
    std::cout << distribution(generator2) << '\n';
}
```

Pick a card

```
import random
random.randint(1,52)
```

Always ask two questions

1. Default seed?
2. Range (), [) or []?

Shuffle the cards first!

```
import random
L = [1, 2, 3, 4, 5]
random.shuffle(L)
# [5, 3, 2, 4, 1]
```

Note that even for small len(x), the total number of permutations of x can quickly grow larger than the period of most random number generators. This implies that most permutations of a long sequence can never be generated. For example, a sequence of length 2080 is the largest that can fit within the period of the Mersenne Twister random number generator.

How to shuffle 3 cards

```
for (int i = 0; i < cards.Length; i++)
{
   int n = rand.Next(cards.Length);
   Swap(ref cards[i], ref cards[n]);
}
```

https://blog.codinghorror.com/the-danger-of-naivete/

- 1 of 3 outcomes after first loop
- 1 of those goes to another of 3
- Finally last goes to another of 3 => 3^3 = 27 possible outcomes
- Should just be 6: 123, 132, 213, 231, 312, 321

A better way…

Knuth-Fisher-Yates shuffle algorithm

```
for (int i = cards.Length - 1; i > 0; i--)
{
  int n = rand.Next(i + 1);
  Swap(ref cards[i], ref cards[n]);
}
```

C++ shuffle kerfuffles

- Lives in the algorithm header
- ~~`std::random_shuffle`~~
  - Might use swap(first[i], first[std::rand() % (i+1)]);
- `std::shuffle` (a deck of cards, maybe)

```
std::random_device rd;
std::mt19937 gen{ rd() };
std::shuffle(deck.begin(), deck.end(), gen);
   //or
std::ranges::shuffle(deck, gen);
```

Choices

- Choices v sample in python

```
things = ['red', 'red', 'red', 'red', 'blue', 'blue']
random.sample(things, k=3)
random.choices(things, k=3)
```

- With or without replacement?
  - Sample: Return a *k* length list of unique elements chosen from the population without replacement.
  - Choice: Return a *k* length list of elements chosen from the population with replacement.

Samples in C++

```
#include <algorithm>
#include <string>
#include <vector>
void sampling()
{
    using namespace std::string_literals;
    std::vector things = {"red"s, "red"s, "red"s, "red"s, "blue"s, "blue"s };
    std::random_device rd;
    std::default_random_engine gen{ rd() };
    std::ranges::shuffle(things, gen);

    std::vector result(things.begin(), things.begin() + 3); // Get the first 3 values
    for (auto thing: result)
        std::cout << thing << '\n';
}
```

But…

- If we want to sample 3 integers out of 1 billion needs a vector with a billion values (memory-inefficient) and then we'd have to shuffle **all of them** (runtime-inefficient)

  https://www.gormanalysis.com/blog/random-numbers-in-cpp/

- Suggests an unordered_set and select by index
- But things have repeats
  - `{"red"s, "red"s, "red"s, "red"s, "blue"s, "blue"s };`

Actually

```
using namespace std::string_literals;
auto things = { "red"s, "red"s, "red"s, "red"s, "blue"s, "blue"s };
std::vector<std::string> result;
std::sample(things.begin(), things.end(),
    std::back_inserter(result), 3,
    std::mt19937{ std::random_device{}() }); // C++17
// or
std::ranges::sample(things,
        std::back_inserter(result), 3,
        std::mt19937{ std::random_device{}() }); // C++20

for (auto thing : result)
    std::cout << thing << '\n';
```

R, S

- Knuth’s algorithms
  - R reservoir sampling, sample without replacement, of *k* items from a population of unknown size *n* in a single pass over the items.
  - S randomly sampling n items from a set of M items, with equal probability, where M >= n and M, the number of items is unknown until the end.

Notes

This function may implement selection sampling or reservoir sampling.

Feature-test macro	Value	Std	Comment
__cpp_lib_sample	201603L	(C++17)	std::sample

Choices in C++

```
void choices()
{
    using namespace std::string_literals;
    std::vector things = { "red"s, "red"s, "red"s, "red"s, "blue"s, "blue"s };
    std::vector<double> weights(things.size(), 1.0/things.size());

    std::discrete_distribution<int> distribution(weights.begin(), weights.end());
    std::random_device rd;
    std::default_random_engine gen{ rd() };
    std::vector<std::string> result(3);
    std::generate(result.begin(), result.end(), [&]{ return things[distribution(gen)]; });

    for (auto thing : result)
        std::cout << thing << '\n';
}
```

How to test

- A dice game?
- Picking a card?
- Shuffling a card?

YOU DO NOT NEED TO TEST YOUR LANGUAGE'S RANDOM NUMBER GENERATOR

BUT DO CHECK YOU ARE USING THE RANDOM NUMBER GENERATOR CORRECTLY

Testing

- Known seed v mock
- Send in 0? (Makes much of maths/simulation study easy)

Seeds

- A seed will give you a known sequence (on a compiler, platform, etc)
- Will it cover all the transitions?
- Fake out the random
  - Try return zero

- Have changes (win/lose) in a separate function to random
- What are you trying to test?

It's not as simple as using a predictable sequence to test it, because even simple changes to the code may use up the sequence in a different order and break tons of tests (which isn't helpful, because all that stuff didn't break).

I know I don't want to go change 50 tests because I reordered two lines.

My suggestion is that somewhere, your code should be *doing* something with the numbers.

You can test whether or not it does the right things with the right numbers

http://wiki.c2.com/?UnitTestingRandomness

Testing, testing

- https://stackoverflow.com/questions/61047296/how-to-replace-the-call-to-random-randint-in-a-function-tested-with-pytest

```
import random
...
def hit(self, enemy, attack):
    dmg = random.randint(self.weapon.damage_min,
                         self.weapon.damage_max) +
                         self.strength // 4
    …
```

Suggested approach

```
def test_player_hit_missed(monkeypatch,
        monster,
        hero):
    monkeypatch.setattr('random.randint',
            lambda a, b: -3)
    hero.hit(monster, 'Scream')
    assert monster.life == 55
```

What do you think?

- Send in values? (and `hit` does the hit)

Testing, … what?

- https://stackoverflow.com/questions/42788644/how-to-test-random-choice-in-python

```
from random import shuffle

def getMaxIndices(lst):
    '''
    Return indices of max value. If max value appears more than once,
    we chose one of its indices randomly.
    '''
    index_lst = [(i, j) for i, j in enumerate(lst)]
    shuffle(index_lst)
    index_lst.sort(key=lambda x: x[1])
    max_index = index_lst.pop()[0]
    return max_index
```

Suggested approach

```
@patch('random.shuffle', lambda x: x)
def test_get_max_Indices():
    max_index = getMaxIndices([4,5,6,7,8])
    assert max_index == 4
```

Alternatives?

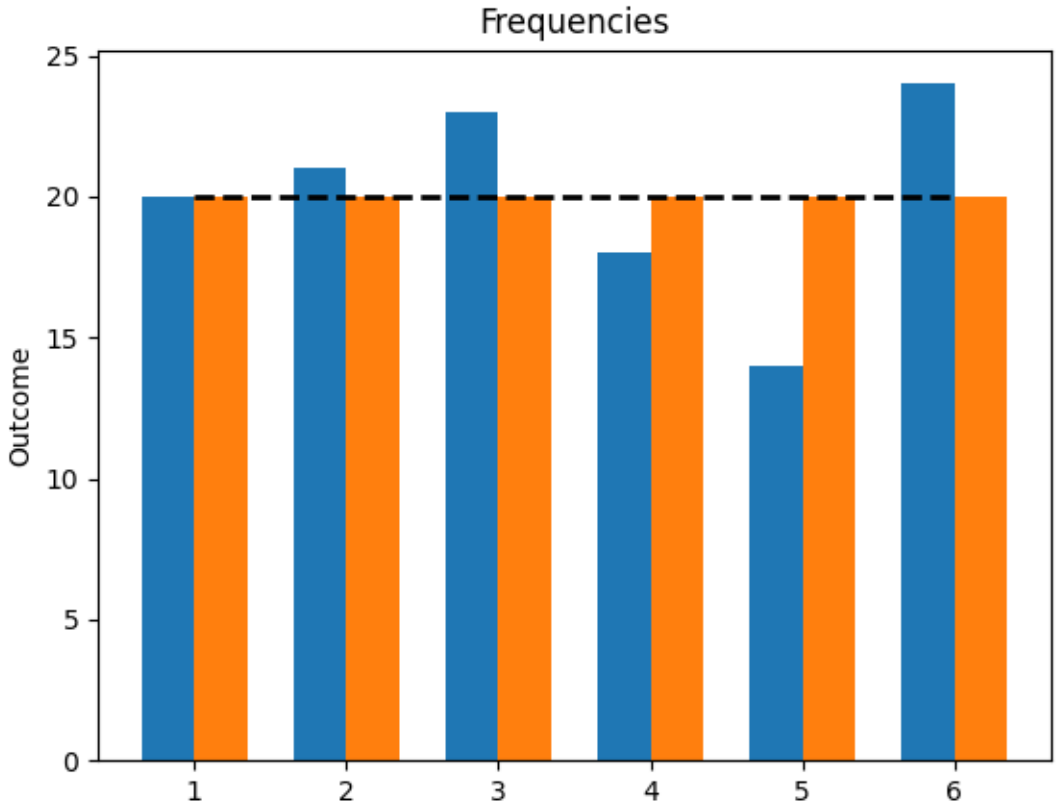
- Deterministic tests - verify code
  - Send in values? (and `hit` does the hit)
  - Never forget edge cases
- Random sequences have properties
  - What average do you expect?
  - Is your uniform distribution giving uniform results?
- Non-deterministic test - find bugs
  - Property based testing
  - Fuzzing
- Also, have you ever run your unit/integration/etc test in random order
  - Random is useful

Beyond uniform discrete

- Dice, cards,…
  - whole numbers (discrete)
  - Equally likely (uniform)
- Continuous (doubles etc)
  - https://www.thusspakeak.com/ak/2014/06/01-WhatAreTheChancesOfThat.html
- Non-uniform
  - Normal
  - Weighted
- Distributions of angles and directional statistics

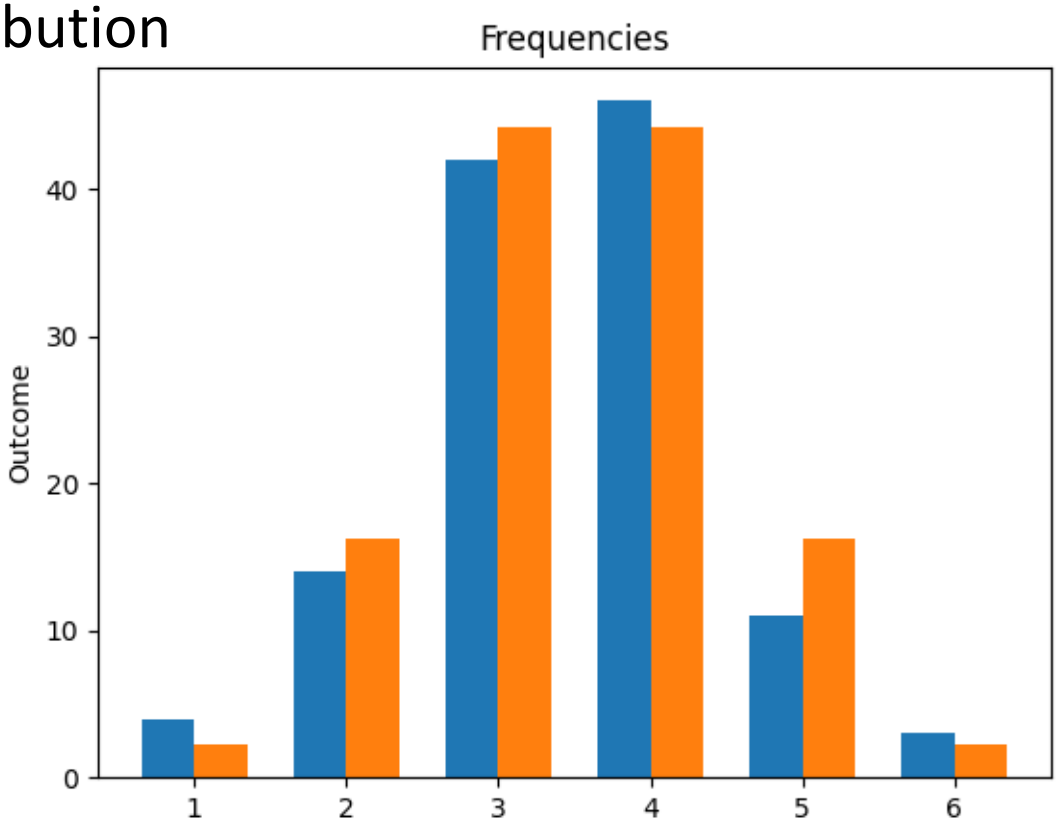
Uniform Distribution x ~ U(a, b)

- x is drawn from the uniform distribution with range [a, b]
- Roll a die 120 times,
  - how many of each number?
  - 1: 20,
  - 2:21,
  - 3:23,
  - 4:18,
  - 5:14,
  - 6:24



Normal Distribution X ~N(3.5, 1)

- The Bell curve or Gaussian distribution
- Bucket continuous numbers
- 1: 4,
- 2: 14,
- 3: 42,
- 4: 46,
- 5: 11,
- 6: 3

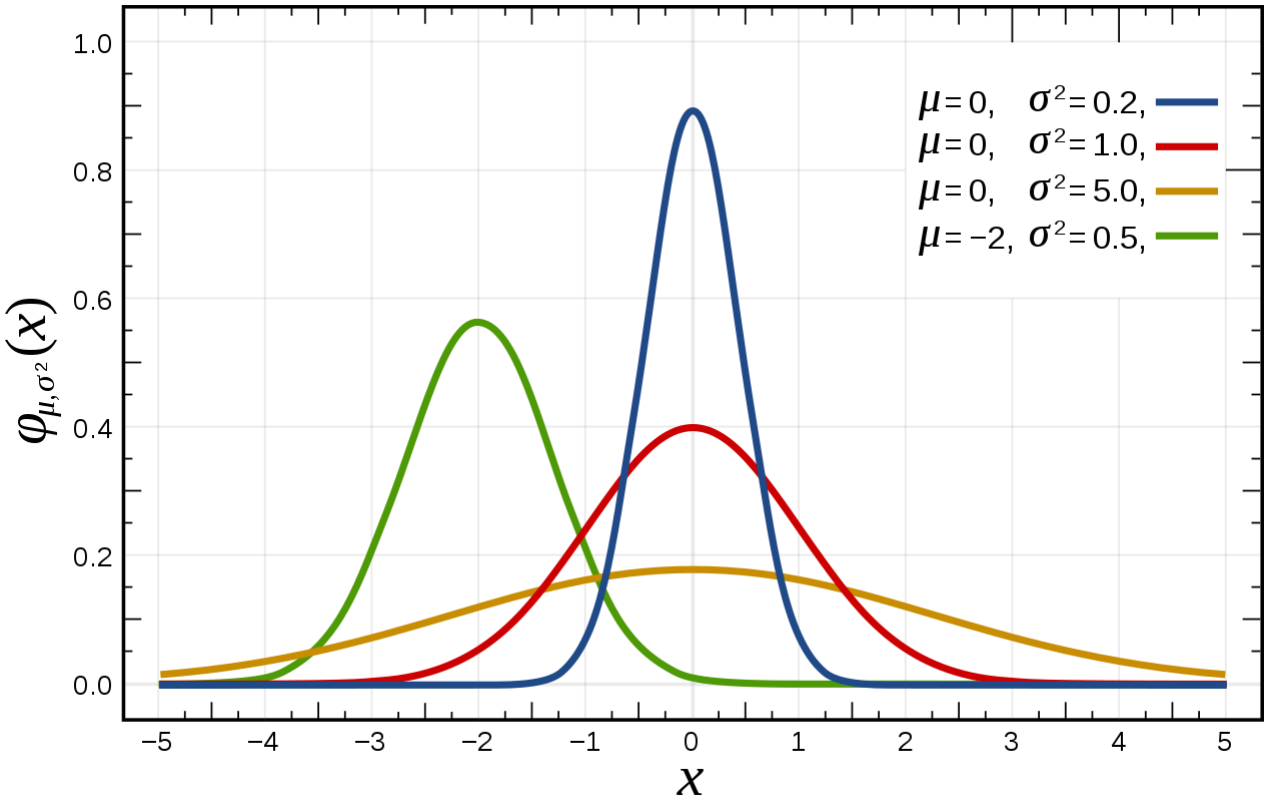


Gaussian pdf

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

μ mean

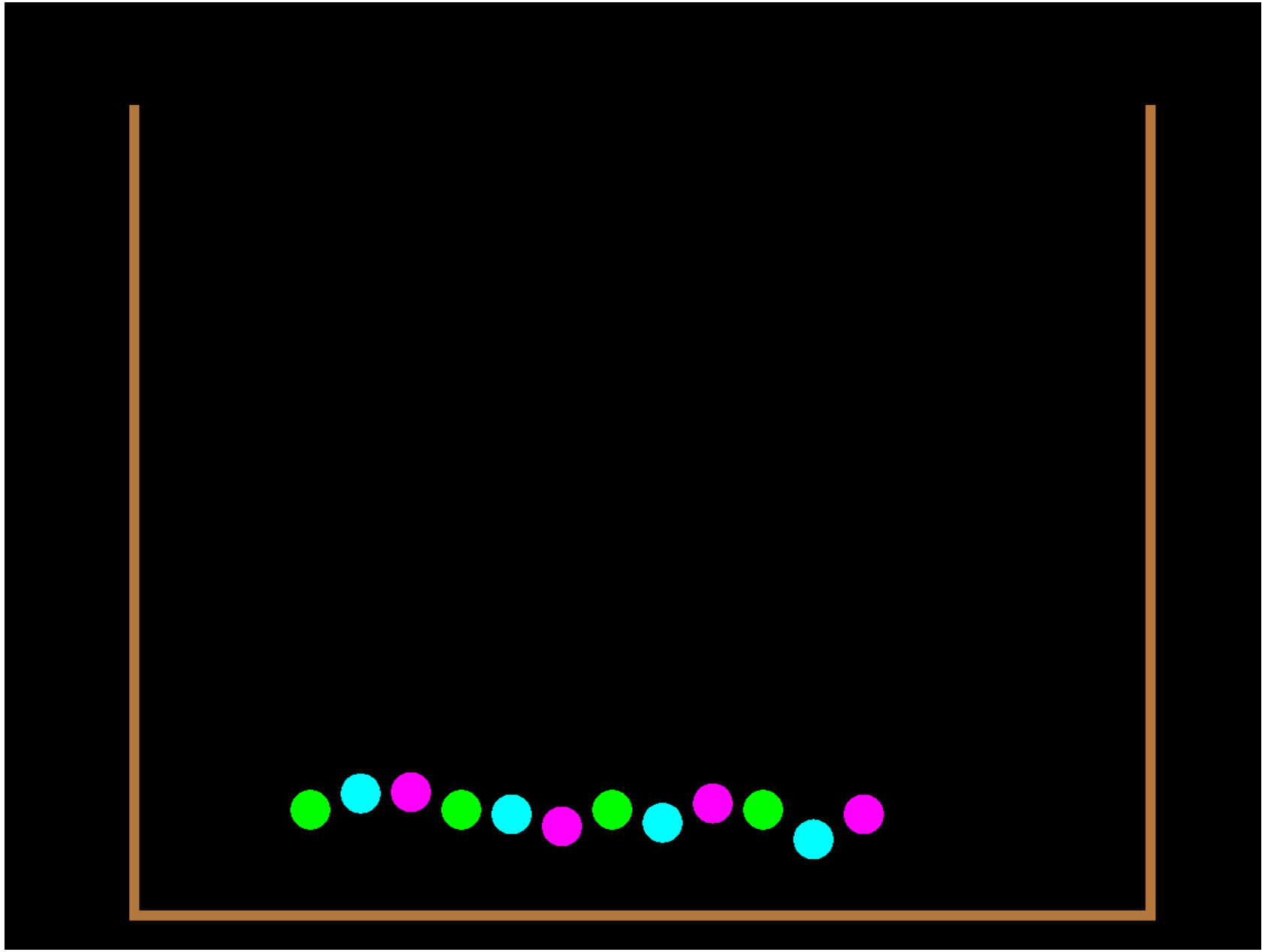
σ standard deviation

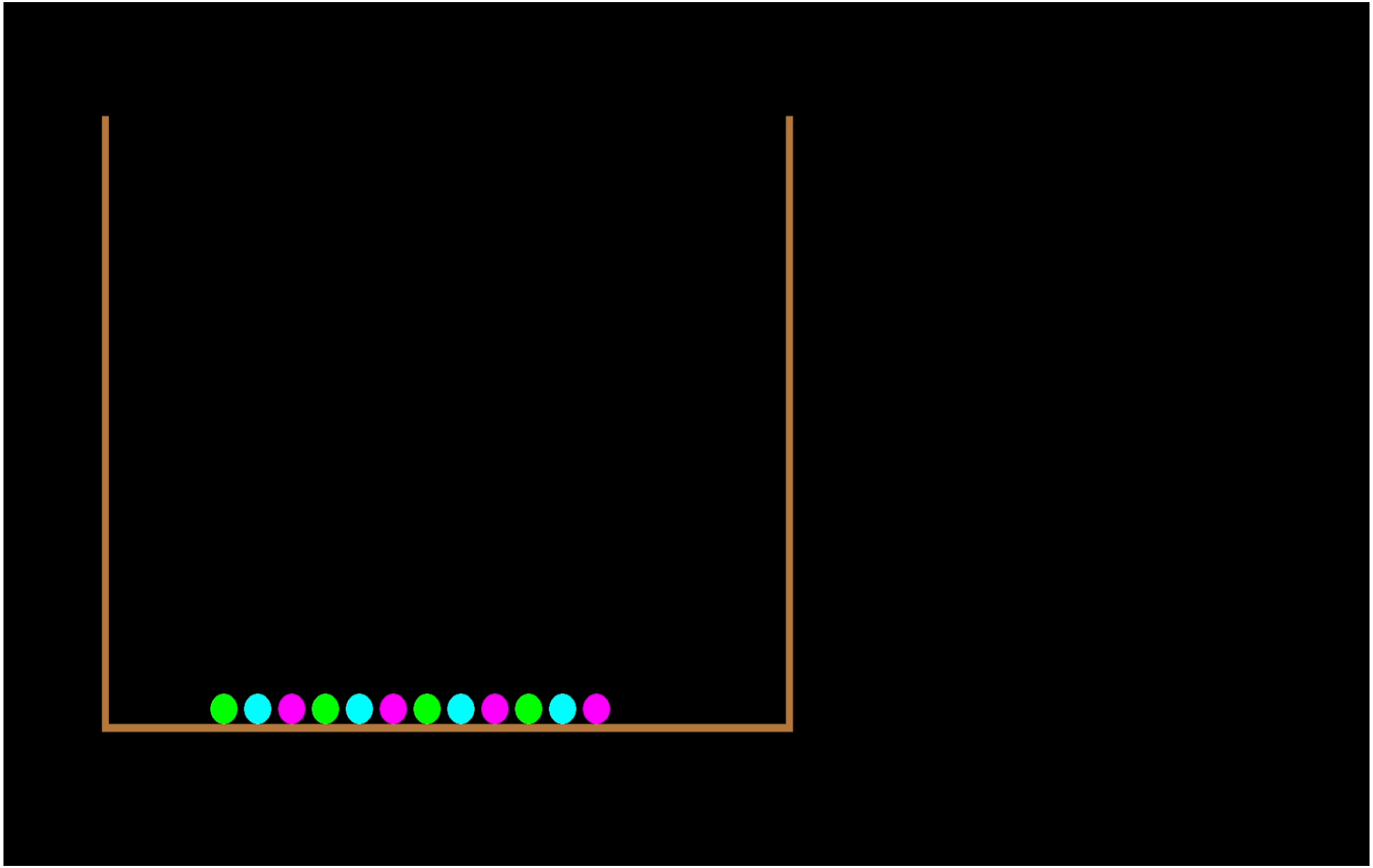


By Inductiveload - self-made, Mathematica, Inkscape, Public Domain, https://commons.wikimedia.org/w/index.php?curid=3817954

Race!

- Two distributions and the steppers
  - Stepper, y+= 20.0f; green blob
  - Uniform, on a float [0, 40]; cyan blob
  - Normal, N~(20.0f, 10.0f); magenta blob
- Who will win?





Formal tests for randomness

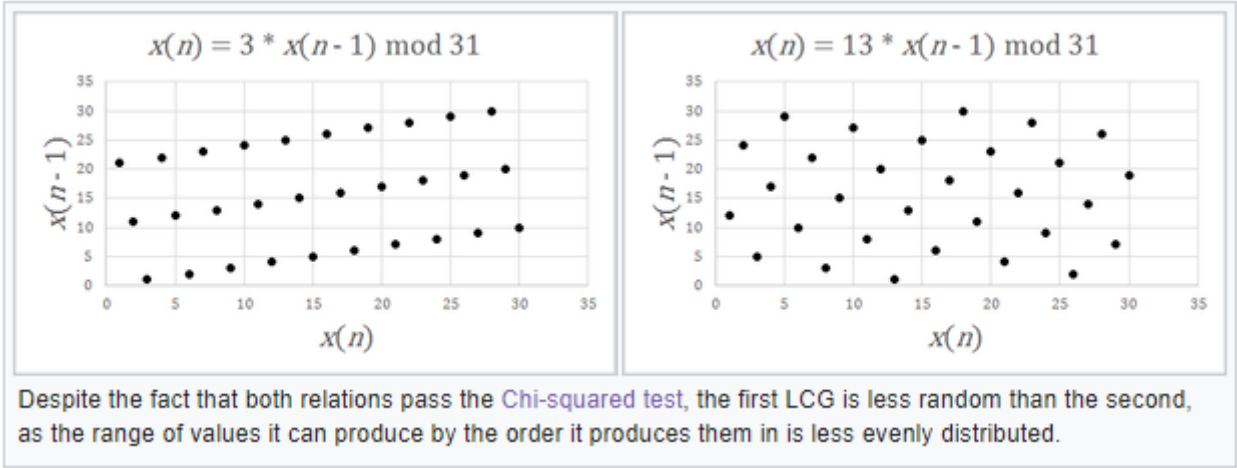
- Does the sequence have a recognisable pattern?
- Can you get all the numbers in the range?...
- Diehard, dieharder

From IMDB
https://www.imdb.com/title/tt0095016/

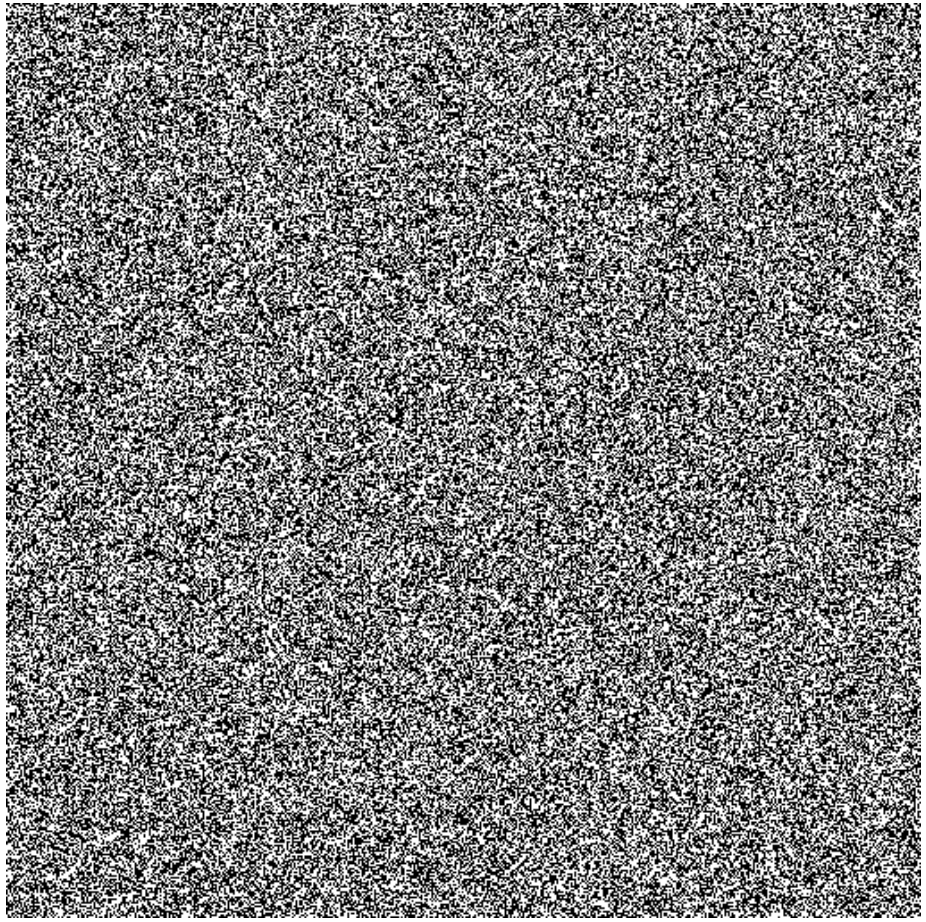
Spectral characteristics

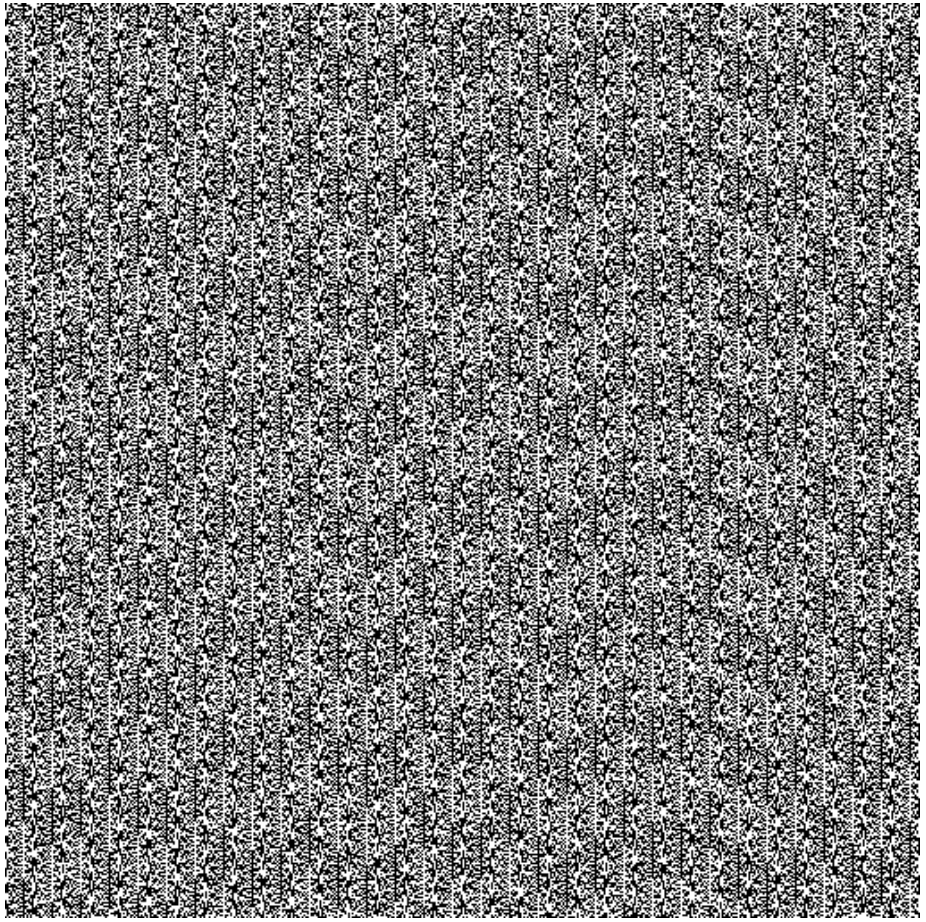
- Looking for repetitive patterns that are near each other
- Plotting x(n) against x(n-1)

https://en.wikipedia.org/wiki/Spectral_test



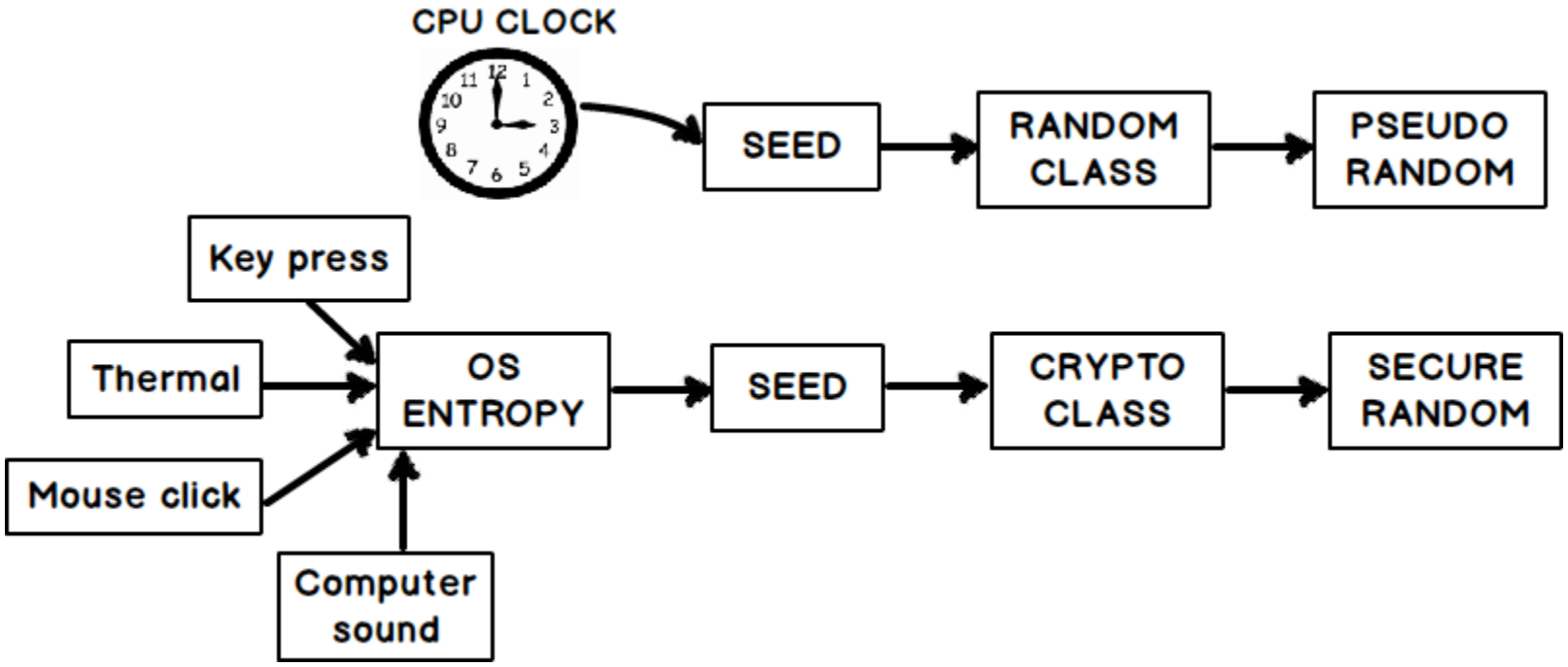
Despite the fact that both relations pass the Chi-squared test, the first LCG is less random than the second, as the range of values it can produce by the order it produces them in is less evenly distributed.





https://www.random.org/analysis/

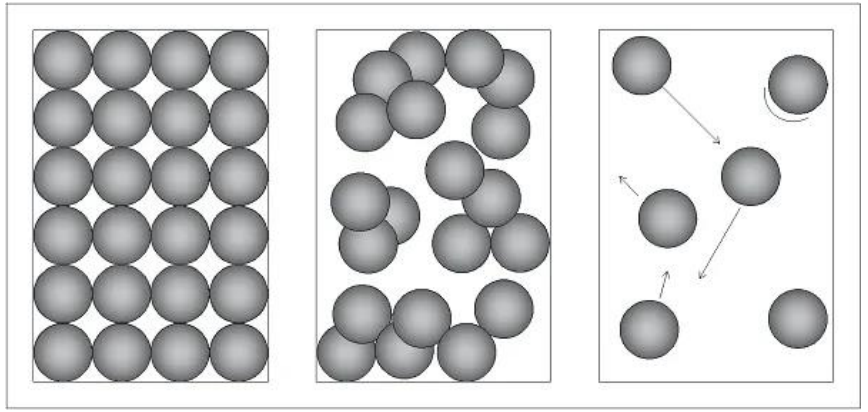
Pseudo v secure



From https://stackoverflow.com/questions/2706500/how-do-i-generate-a-random-integer-in-c

Entropy

- The number of possible microscopic arrangements or states of individual atoms and molecules of a system that comply with the macroscopic condition of the system. (Ludwig Boltzmann)
- The average level of "information", "surprise", or "uncertainty" inherent to the variable's possible outcomes. (Claude Shannon)



Information Entropy

$$H(X) = -\sum_{x \in X} p(x) log p(x)$$

- Fair coin, two outcomes, p(head)=p(tail)=0.5
  - H(X) = $-({}^{1}/_{2}\log({}^{1}/_{2}) + {}^{1}/_{2}\log({}^{1}/_{2})) = -({}^{1}/_{2} \times -1 + {}^{1}/_{2} \times -1) = 1$
- Unfair coin, two outcomes, p(head)=0, p(tail)=1
  - H(X) = $-(0 \times \log(0) + 1 \times \log(1)) = -(0 + {}^{1}/_{2} \times 0) = 0$

0: No surprise

1: Total surprise

Entropy

“I've seen winzip used as a tool to measure the randomness of a file of values before (obviously, the smaller it can compress the file the less random it is).”

http://wiki.c2.com/?UnitTestingRandomness

Cryptography

- NIST “provides guidelines and recommendations for generating random numbers for cryptographic use”
  - https://csrc.nist.gov/projects/random-bit-generation
- Also goes wrong
  - E.g. SSL, Key generation, RSA public key factoring

https://en.wikipedia.org/wiki/Random_number_generator_attack

- “the elliptic curve digital signature algorithm – ECDSA – demands that the random number used to sign a private key is only ever used once. If the random number generator is used twice, the private key is recoverable.”

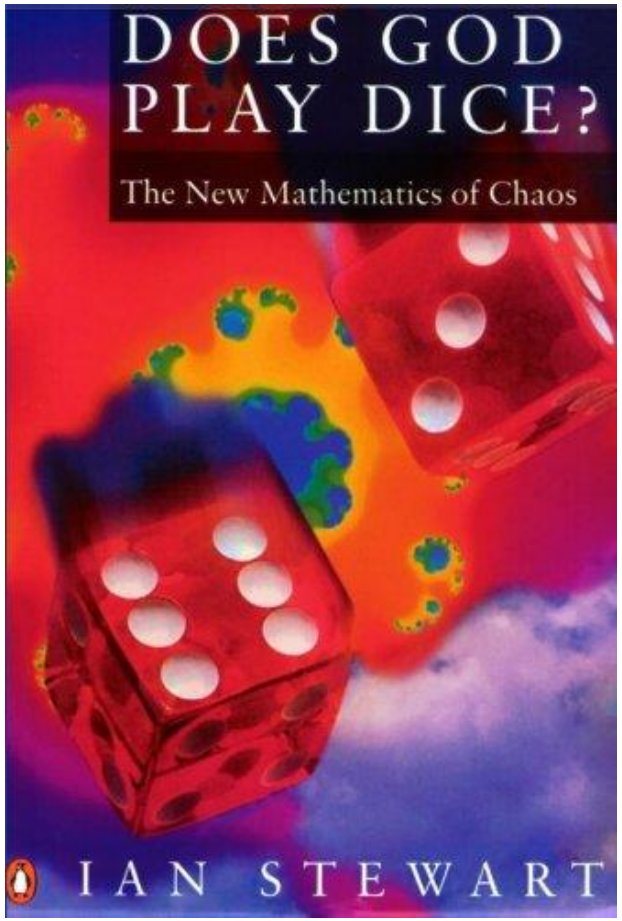
https://www.theregister.com/2013/08/12/android_bug_batters_bitcoin_wallets/

  - The Java class SecureRandom (used by the vulnerable wallets) can generate collisions for the value r. (See http://armoredbarista.blogspot.com/2013/03/randomly-failed-weaknesses-in-java.html)

Integrated balanced homodyne detector

- 100-Gbit/s Integrated Quantum Random Number Generator Based on Vacuum Fluctuations
- “Quantum random number generation allows for the creation of truly unpredictable numbers due to the inherent randomness available in quantum mechanics.”

  https://journals.aps.org/prxquantum/abstract/10.1103/PRXQuantum.4.010330


DOES GOD
PLAY DICE?
The New Mathematics of Chaos
IAN STEWART

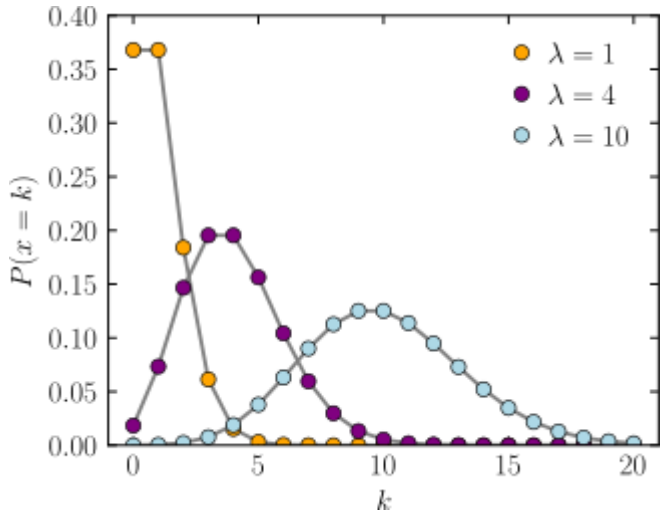
What have we learnt?

- There’s no such thing as a random number
- C++ gives us choices
- Things change, so keep learning
- Seemingly arbitrary outcomes make for good games
  - And some useful stuff too
  - And fun
  - Time for one more demo?

Jumpers

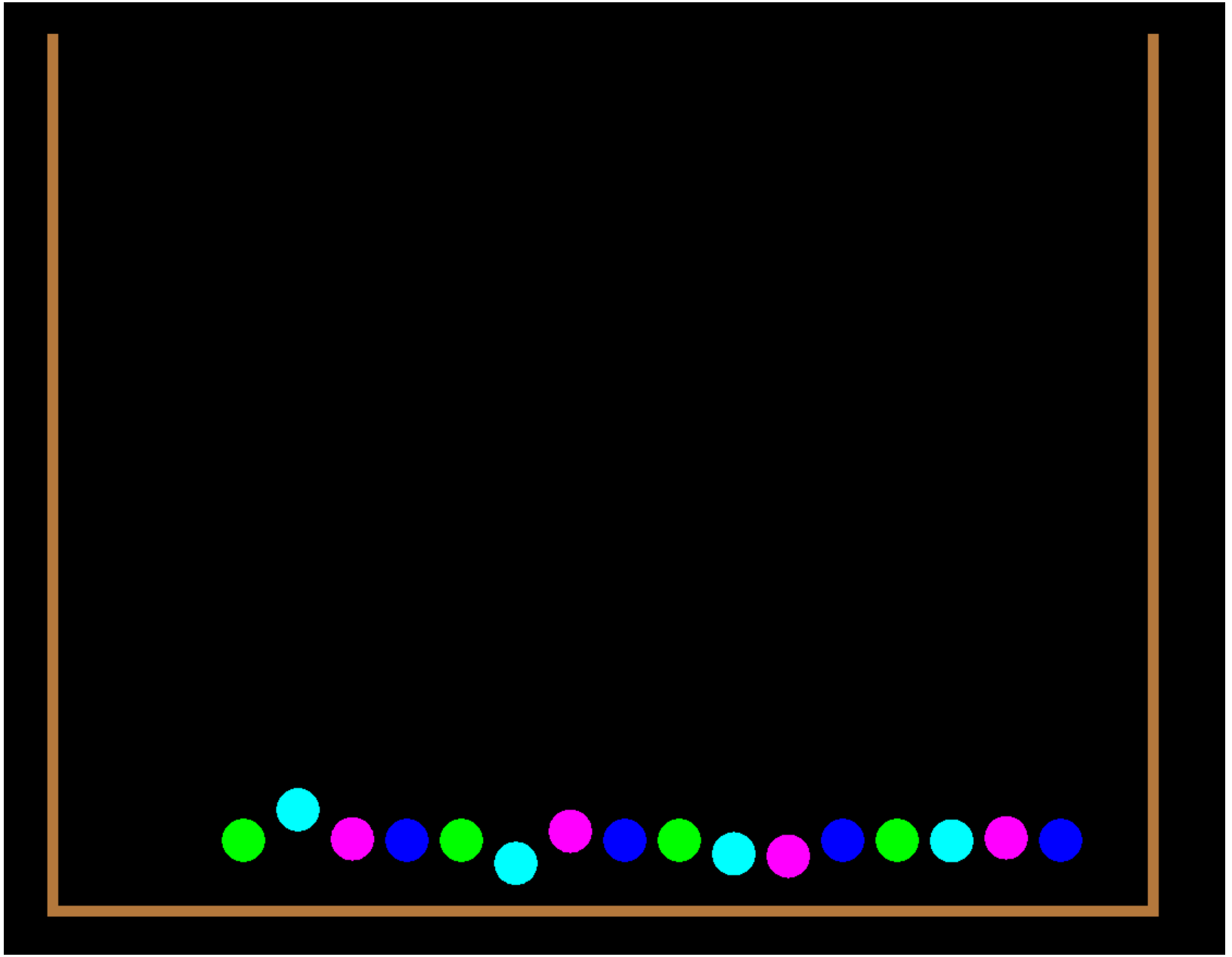
```
std::poisson_distribution<> distribution(0.03125); // 1 in 32
    int jump = distribution(generator);
    y += 20 + jump * 40.;
```

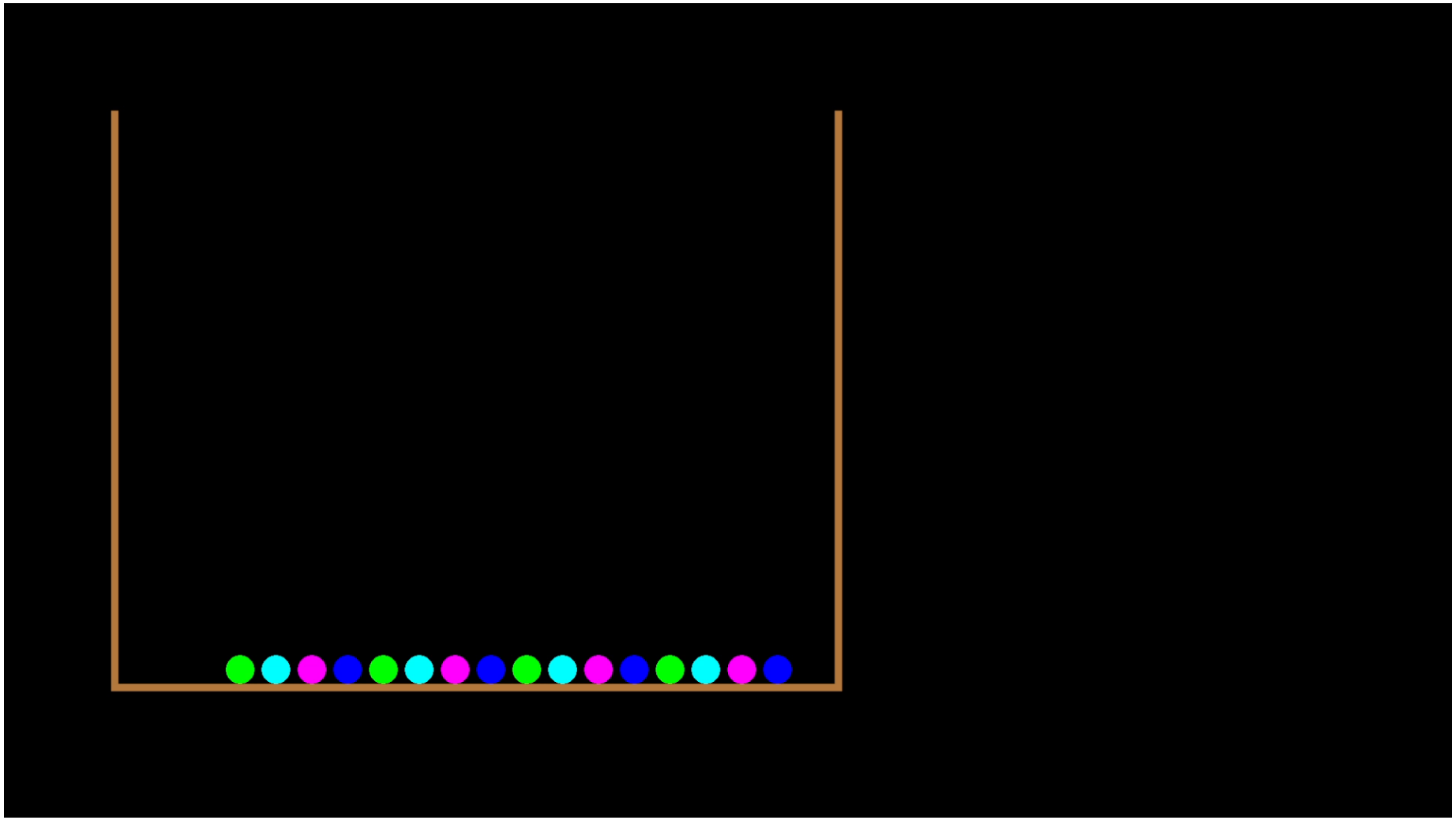
- 3 distributions and the steppers
  - Stepper, y+= 20.0f; green blob
  - Uniform, on a float [0, 40]; cyan blob
  - Normal, N~(20.0f, 10.0f); magenta blob
  - Jumper, Poisson, lambda 1/32; blue blob



From https://en.wikipedia.org/wiki/Poisson_distribution

>Race\Debug\SFMLRace.exe j

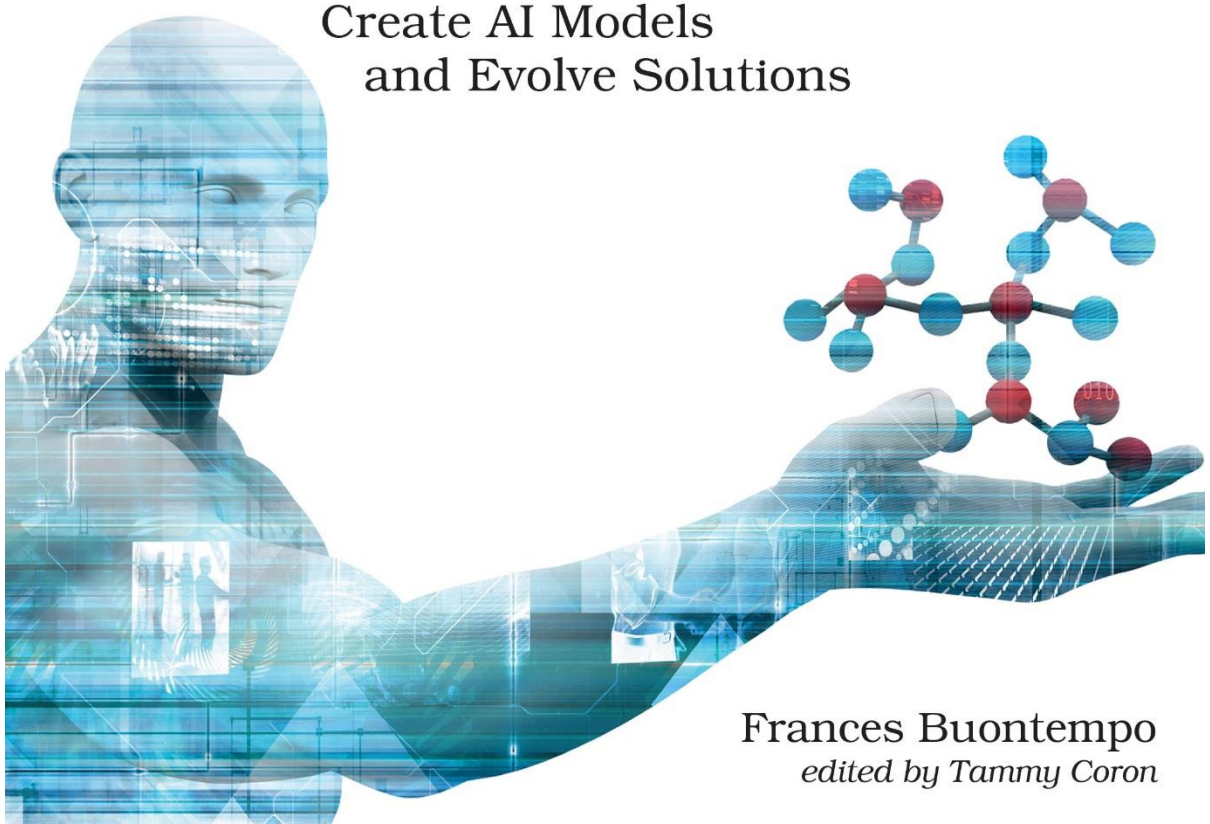




The Pragmatic Programmers

Genetic Algorithms and Machine Learning for Programmers

Create AI Models and Evolve Solutions



Frances Buontempo
edited by Tammy Coron

C++ Book

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https://www.manning.com/books/c-plus-plus-bookcamp