Disclaimer

This isn’t your usual C++ talk! It contains

- Math
- Physics
- Mentions your math and physics teacher
- C++ Code

All of this is for the greater good!

- Comprehensive solution for physical units in matrices
- Even stronger type-safety in C++

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SW developer at Bosch (since 2008)

- Focus on solving real-world problems using C++
- Object tracking framework for self-driving car projects
- Author and maintainer of *type_safe_matrix* library

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PHYSICAL UNITS FOR MATRICES:

HOW HARD CAN IT BE?

PHYSICAL UNITS FOR MATRICES:

HOW HAR~~D~~* CAN IT BE?
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*GEORGE W. HART, MULTIDIMENSIONAL ANALYSIS

Background

C++ in the automotive industry / at Bosch

- Development processes with lots of code reviews
- Code should be written for the reader
  - C++ types that express the content
  - Leverage C++'s type system to catch problems early
- (in-house) physical units library (for scalars) in use for over 15 years
- (in-house) linear algebra library for vector and matrix arithmetic

=> Missing: general solution for physical units in linear algebra types

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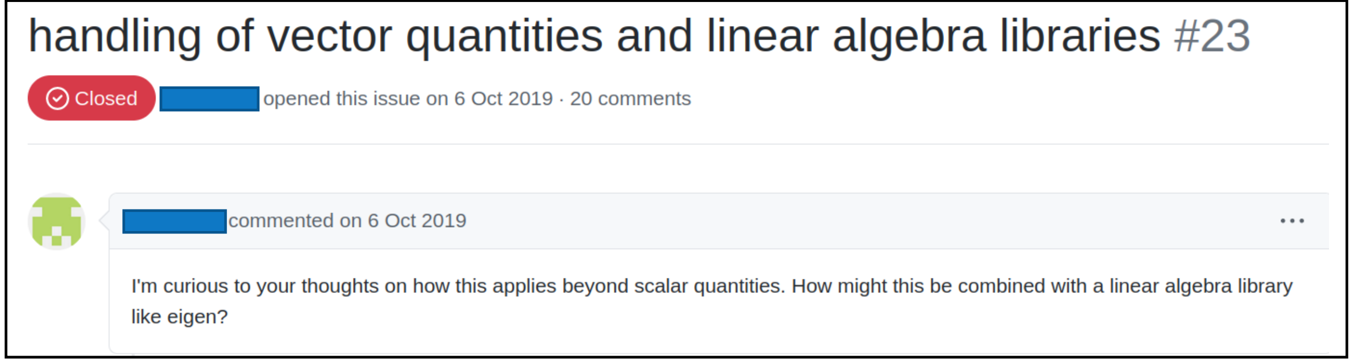
Background

C++ open-source libraries and recent proposals for the standard library

- Linear algebra: Eigen, blaze and http://wg21.link/P1385
- Physical units: https://github.com/mpusz/units (http://wg21.link/P1935), boost units


handling of vector quantities and linear algebra libraries #23

Closed opened this issue on 6 Oct 2019 · 20 comments

commented on 6 Oct 2019

I'm curious to your thoughts on how this applies beyond scalar quantities. How might this be combined with a linear algebra library like eigen?

```
fs_vector<si::length<si::metre>, 3> v = { 1 * m, 2 * m, 3 * m };
fs_vector<si::length<si::metre>, 3> u = { 3 * m, 2 * m, 1 * m };
std::cout << "v + u = " << v + u << "\n";  // [4m, 4m, 4m]
```

=> Missing: general solution for physical units in linear algebra types

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Further motivation

P1417: Historical lessons for C++ linear algebra library standardization

“Linear algebra libraries all must figure out what to do if users attempt to perform operations on objects with incompatible dimensions. […] This gets more complicated, though, if we generalize “compatible dimensions” to the mathematical idea of a “vector space.” Two vectors might have the same dimensions, but it still might not make sense to add them together. For example, I can’t add a coordinate in 3-D Euclidean space to a quadratic polynomial with real coefficients, just like I can’t add meters to seconds.“ [Sec 3.6]

”Expression templates may hinder use of auto […] because the type […] may be some expression type that may hold references to concrete linear algebra objects. Returning the expression may result in dangling references“ [Sec 3.1]

http://wg21.link/P1417

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Non-expressive code examples

Can you tell what this line of code does?

```
measurement_vector[2] = other_vector[3];
```

- What do the vector entries describe?
- Is this an out-of-bounds access?

2nd try:

```
measurement_vector[VELOCITY_X] = other_vector[POSITION_X];
```

- Have the right index constants for the vector type been used?
- Is assigning a position to a velocity really intended?

3rd try:

```
measurement_vector[VELOCITY_X] = other_vector[VELOCITY_X];
```

- In which coordinate frame are measurement_vector and other_vector?

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What we ideally want

My linear algebra library wishlist:

- Protection against out-of-bounds access (compile-time please!)
- Expressive (and enforced) names for vector / matrix entries

- Support of *non-uniform* physical units in vectors / matrices
- Compatibility check for physical units during all matrix operations (compile-time)
- Coordinate frame annotation for vectors and transformations

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Named index structs

Identify each entry with a unique name: **DistanceX_C**

```
struct DX_C :
CartesianIdxType<VehicleFrontAxleCoords,
                 tsm::CartesianXAxis,
                 si::Metre>{};
```

coordinate frame
axis identifier
si unit template

Physical quantity | Axis | Coordinate frame

```
DistanceX_SENSOR_C
DistanceY_SENSOR_C
VelocityX_SENSOR_C
VelocityY_SENSOR_C
Acceler.X_SENSOR_C
Acceler.Y_SENSOR_C
```

```
struct DY_SENSOR_C :
CartesianIdxType<SensorCoords,
                 tsm::CartesianYAxis,
                 si::Metre>{};
```

```
struct VehicleFrontAxleCoords : public CoordinateSystem<si::Metre> {
  using Moving = std::false_type; // is the frame moving wrt to the “fixed” earth frame?
};
```


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One type for almost everything

```
template<class Scalar, class RowIdxList, class ColIdxList, class MatrixTag>
class TypeSafeMatrix {
  ... // methods
  private:
    Eigen::Matrix<Scalar, SizeOf<RowIdxList>::value, SizeOf<ColIdxList>::value> m_matrix;
};
```

```
template<class Scalar, class RowIdxList, class MatrixTag>
using TypeSafeVector = TypeSafeMatrix<Scalar,
                                      RowIdxList,
                                      tsm::TypeList<tsm::NoIdxType>,
                                      MatrixTag>;
```

```
template<class Scalar>
using PosVec3InVehicleFrame<Scalar> =
  tsm::TypeSafeVector<Scalar, tsm::TypeList<DX_C, DY_C, DZ_C>, tsm::VectorTag>;
```


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Creating a vector and access to elements

```
PosVec3InVehicleFrame<double> pos_vehicle{
    other_position.entry<DX_C>(),                    // copy 1 entry from other vector  (***)
    tsm::wrapCoeffSi<DY_C>(si::Metre<double>{1.}), // unit argument                    (**)
    tsm::wrapCoeff<DZ_C>(3.)};                       // plain scalar argument            (*)
```

```
// full check, only the same index from the same vector type can be assigned
pos_vehicle.assignEntry(other_position.entry<DX_C>());                          // (***)
```

```
// unit check, allows assigning a y position from a different frame
pos_sensor.coeffSiRef<DX_SENSOR_C>() = pos_vehicle.coeffSi<DY_C>();            // (**)
```

```
// no checks except for scalar type, allows assigning a velocity to a position
pos_vehicle.at<DX_C>() = vel_sensor.at<VY_SENSOR_C>();                          // (*)
```


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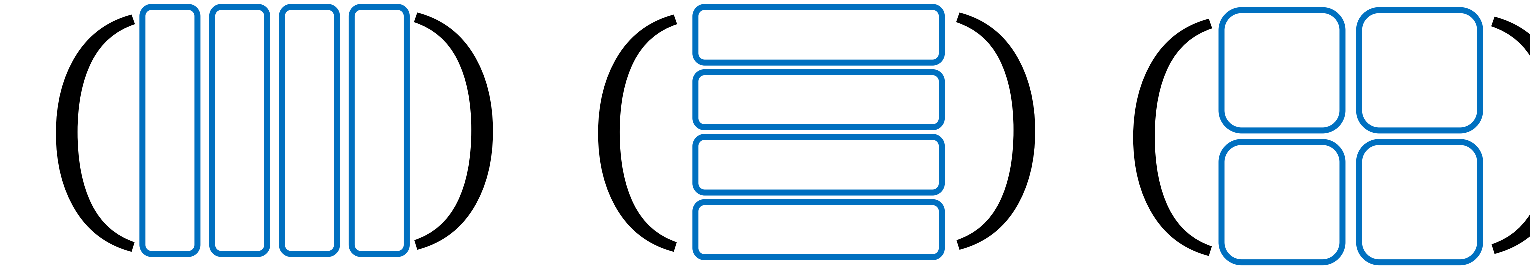
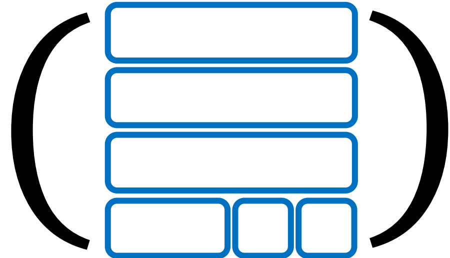
More ways to create a vector / matrix

```
DeltaPosVec3InSensorFrame<double> delta_pos_sensor{
    si::Second<double>{2.0} * other_vel_sensor.head<2>(),  // other 2d vector in same frame
    other_delta_pos.entry<DZ_SENSOR_C>()};
```

```
tsm::TypeSafeMatrix<double, tsm::TypeList<DX_C, VX_C>, tsm::TypeList<DX_C, VX_C>,
                    tsm::CovarianceMatrixTag> covariance{
    tsm::wrapCoeffSi<DX_C, DX_C>{...}, tsm::wrapCoeffSi<DX_C, VX_C>{...},
    tsm::wrapCoeffSi<VX_C, DX_C>{...}, tsm::wrapCoeffSi<VX_C, VX_C>{...}
};
```





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Matrix element access

Declare a type-safe (jacobian) matrix

```
tsm::TypeSafeMatrix<double,
        tsm::TypeList<MEAS_DR, // radial dist[m]
                      MEAS_VR, // radial vel[m/s]
                      MEAS_ANGLE>, // angle [rad]
        tsm::TypeList<DX_C,    // distance x [m]
                      DY_C,    // distance y [m]
                      DZ_C>,   // distance z [m]
        tsm::JacobianMatrixTag> jacobian{...};
```

Jacobian matrix	DX_C [m]	DY_C [m]	DZ_C [m]
MEAS_DR [m]			
MEAS_VR [m/s]			
MEAS_ANGLE [rad]			

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Matrix element access

Which physical unit should be returned?

```
??? value = jacobian.coeffSi<MEAS_VR, DY_C>();
```

Jacobian matrix	DX_C [m]	DY_C [m]	DZ_C [m]
MEAS_DR [m]			
MEAS_VR [m/s]		???	
MEAS_ANGLE [rad]			

Column exponent = -1

Row exponent = 1

Jacobian matrix	DX_C [m]	DY_C [m]	DZ_C [m]
[m]			
[m/s]		[m/s]^row_exp * [m] ^col_exp	
[rad]			

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Taxonomy of vector and matrix types

Matrix / vector type	Row exponent	Column exponent	Nr of columns
Covariance matrix	1	1	
Jacobian matrix	1	-1	
Information matrix	-1	-1	
Position vector (VectorTag)	1	0	1
Position vector collection	1	0	>1
Displacement vector (DeltaVectorTag)	1	0	1
Displacement vector collection	1	0	>1
Information vector	-1	0	1
Information vector collection	-1	0	>1

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Now we have unit-safe
element access and out-of-bounds protection

Can we do more???

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Is unit-safety enough?

Let’s try being unit-safe for all operations

Should this operation be allowed?

$$\begin{pmatrix} DX_C[m] \\ DY_C[m] \end{pmatrix} + \begin{pmatrix} VX_C[m/s] \\ VY_C[m/s] \end{pmatrix}$$

Should this be allowed?

$$\begin{pmatrix} [m] \\ [m] \end{pmatrix} + \Delta \begin{pmatrix} [m] \\ [m] \end{pmatrix}$$

Really?

Index types do not match!

$$\begin{pmatrix} DX_C \\ DY_C \end{pmatrix} + \Delta \begin{pmatrix} DY_C \\ DZ_C \end{pmatrix}$$

Unit-safety is not enough,
we need index-type safety!

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Matrix multiplication

What happens when e.g. transforming a covariance matrix to another frame?

```
cov_sensor = jacobian * cov_vehicle * jacobian.transpose();
```

Jac	DX_C	DY_C	VX_C	VY_C
	-1			
DX_{SEN}	1.0	0	0	0
DX_{SEN}	0	1.0	0	0
VX_{SEN}	0	0	1.0	0
VY_{SEN}	0	0	0	1.0

1

* 1 1

Cov	DX_C	DY_C	VX_C	VY_C
	1			
DX_C	3.1	0	0	0
DY_C	0	2.4	0	0
VX_C	0	0	8.5	0
VY_C	0	0	0	6.4

* -1

Jac^T	DX_{SEN}	DY_S	VX_S	VY_{SEN}
	1			
DX_C	1.0	0	0	0
DY_C	0	1.0	0	0
VX_C	0	0	1.0	0
VY_C	0	0	0	1.0

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Important classes

Building blocks:

```
template<class Derived> class MatrixBase{};
```

```
template<class Promotion,    // infers resulting TypeSafeMatrix
         class LinalgExpression> // Eigen expression
class MatrixExpression : public
  MatrixBase<MatrixExpression<Promotion, LinalgExpression>>{};
```

```
template<class Scalar, class RowIdxList, class ColIdxList, class MatrixTag>
class TypeSafeMatrix : public MatrixBase<
    TypeSafeMatrix<ScalarT, RowIdxList, ColIdxList, MatrixTagT>>{};
```


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What operator+ looks like

```
template<typename OtherLeaf_T>
auto operator+(const MatrixBase<OtherLeaf_T>& other) const
    -> detail::MatrixExpression<detail::Promotion2dAddition<Leaf_T, OtherLeaf_T>,
                                decltype(this->underlying() + other.underlying())> {…}
```

```
template<typename Expr1, typename Expr2>
struct Promotion2dAddition
    : RequiresIdenticalScalarType<Expr1, Expr2>,                    // inject Scalar type
      RequiresIdenticalRowIndices<Expr1, Expr2>,                    // inject RowIdxList type
      RequiresIdenticalColIndices<Expr1, Expr2>,                    // inject ColIdxList type
      RequiresIdenticalRowUnitExponent<Expr1, Expr2>,
      RequiresIdenticalColUnitExponent<Expr1, Expr2>,
      RequiresMatrixTagsAdditionCompatible<typename Expr1::MatrixTag,   // inject MatrixTag type
                                           typename Expr2::MatrixTag> {};
```

```
template<typename Expr1, typename Expr2>
struct RequiresIdenticalRowIndices {
    static_assert(detail::TypeIdentityChecker<typename Expr1::RowIdxList,
                                              typename Expr2::RowIdxList>::value,
                  "Row index types are not equal as required");
    using RowIdxList = typename Expr1::RowIdxList;
};
```


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Compiler error messages

```
auto res = tsm::PosVector3InVehicleFrame{} + tsm::DeltaPosVector2InVehicleFrame{};
```

```
../type_safe_matrix/typelist_operations.h: In instantiation of ‘class
detail::TypeIdentityChecker<TypeList<DX_C, DY_C, DZ_C>, TypeList<DX_C, DY_C> >’:
../type_safe_matrix/promotion_precondition_checks.h:216:35:   required from ‘class
detail::RequiresIdenticalRowIndices<TypeSafeMatrix<double, TypeList<DX_C, DY_C, DZ_C>,
TypeList<NoIdx>, VectorTag>, TypeSafeMatrix<double, TypeList<DX_C, DY_C>, TypeList<NoIdx>,
DeltaVectorTag> >’
../type_safe_matrix/typed_matrix_promotions.h:197:7:   required from ‘class
detail::Promotion2dAddition<TypeSafeMatrix<double, TypeList<DX_C, DY_C, DZ_C>,
TypeList<NoIdx>, VectorTag>, TypeSafeMatrix<double, TypeList<DX_C, DY_C>, TypeList<NoIdx>,
DeltaVectorTag> >’
......
../type_safe_matrix/test/usage_examples.cpp:505:67:   required from here
../type_safe_matrix/typelist_operations.h:66:3: error: static assertion failed: actual type
(1st template arg of TypeIdentityChecker) does not match desired type (2nd arg);
```


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Compiler error message with C++20 concepts

```
auto res = tsm::PosVector3InSensorFrame{} + tsm::DeltaPosVector2InSensorFrame{};
```

```
error: no match for 'operator+' (operand types are 'Vector3' {aka 'TypeSafeMatrix<double,
TypeList<DX_C, DY_C, DZ_C>, TypeList<NoIdxType>, VectorTag>'} and 'Vector2' {aka
'TypeSafeMatrix<double, TypeList<DX_C, DY_C>, TypeList<NoIdxType>, VectorTag>'})
44 |      auto res = Vector3{} + Vector2{};
   |                 ~~~~~~~~~ ^ ~~~~~~~~~
   |                 |           |
   |                 |           TypeSafeMatrix<[...],TypeList<DX_C, DY_C>,[...],[...]>
   |                 TypeSafeMatrix<[...],TypeList<DX_C, DY_C, DZ_C>,[...],[...]>
33 |    TypeSafeMatrix operator+(const OtherT& other) requires Addable<TypeSafeMatrix,
OtherT>       |                        ^~~~~~~~
<source>:33:19: note: constraints not satisfied
required for the satisfaction of 'Addable<TypeSafeMatrix<ScalarT, RowIdxListT, ColIdxListT,
MatrixTagT>, OtherT>‘
note: nested requirement 'is_same_v<typename T1::RowIdxListType, typename
T2::RowIdxListType>' is not satisfied
```


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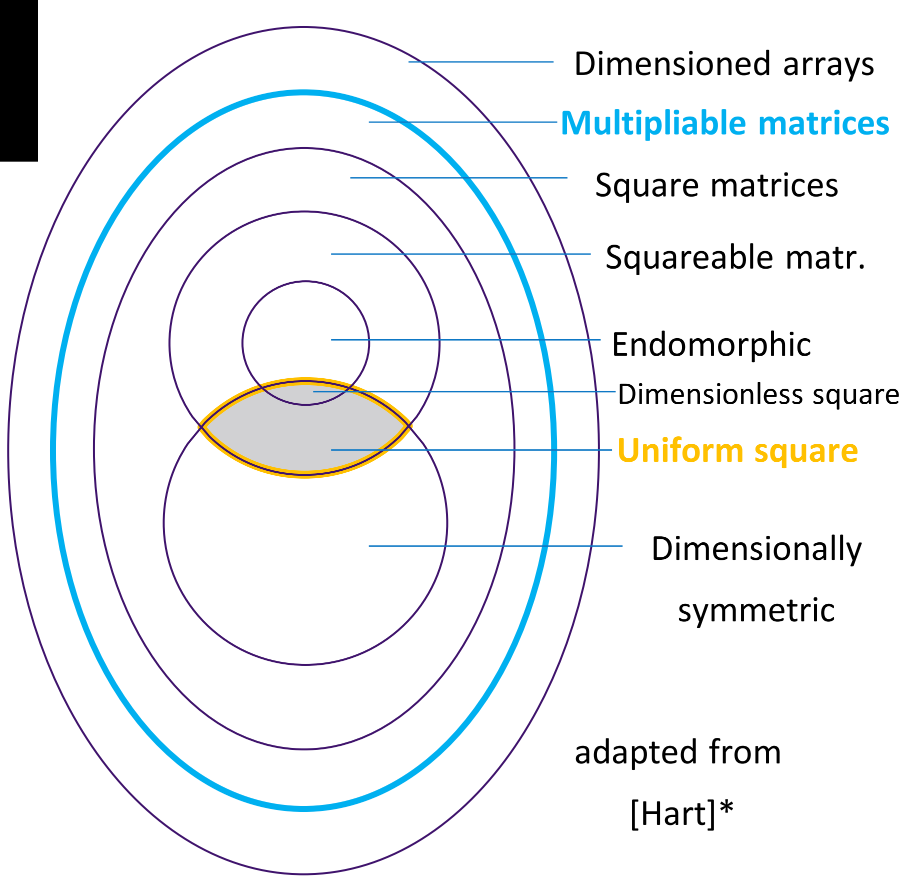
Uniform matrices vs. *Index-oriented design (TypeSafeMatrix)*

```
fs_vector<si::length<si::metre>, 3> v = {1*m, 2*m, 3*m};
fs_vector<si::length<si::metre>, 3> u = {3*m, 2*m, 1*m};
std::cout << "v + u = " << v + u << "\n";  // [4m,4m,4m]
```



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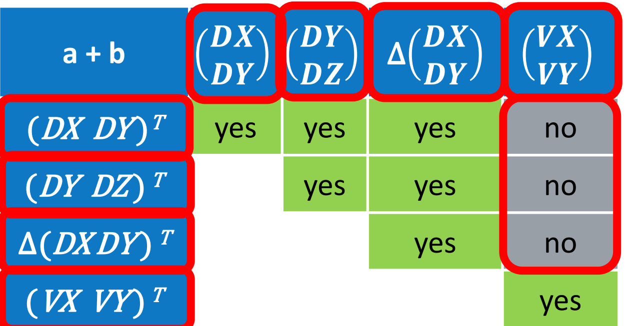
Uniform matrices vs. *Index-oriented design (TypeSafeMatrix)*

What can be added / multiplied?

Unit as value type



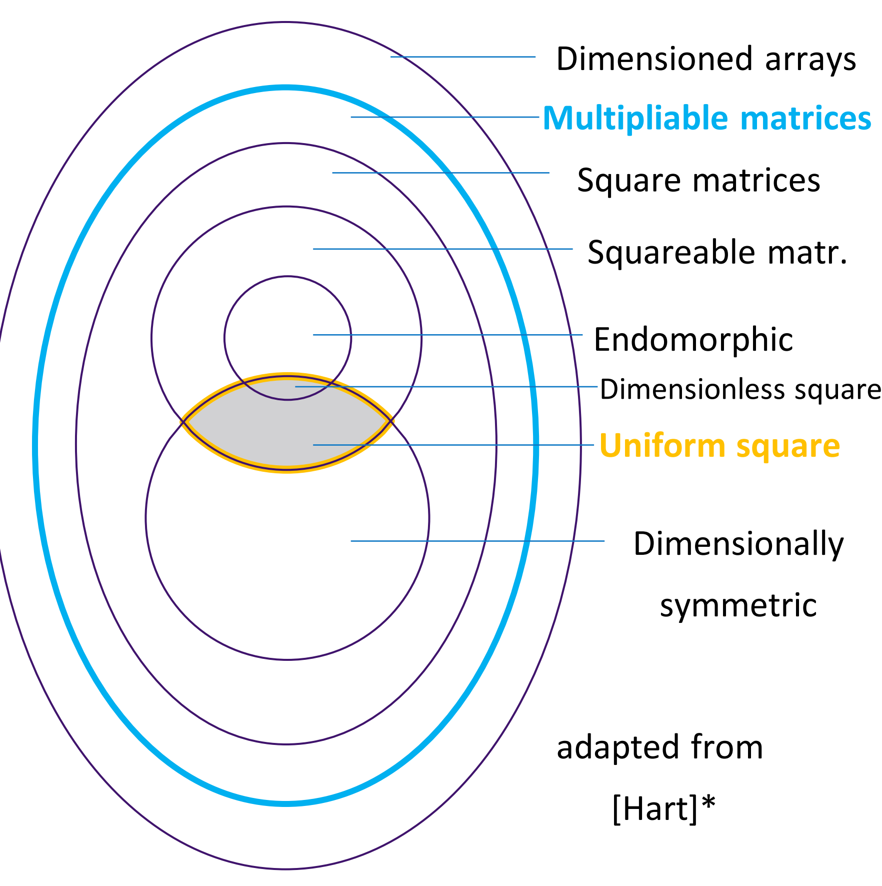
a + b	$\begin{pmatrix}DX\\DY\end{pmatrix}$	$\begin{pmatrix}DY\\DZ\end{pmatrix}$	$\Delta\begin{pmatrix}DX\\DY\end{pmatrix}$	$\begin{pmatrix}VX\\VY\end{pmatrix}$
$(DX\ DY)^T$	yes	yes	yes	no
$(DY\ DZ)^T$		yes	yes	no
$\Delta(DX\,DY)^T$			yes	no
$(VX\ VY)^T$				yes

A * B	jac	jac^T	cov	vector
jac	yes	yes	yes	yes
jac^T	yes	yes	yes	yes
cov	yes	yes	yes	yes
vector	yes	yes	yes	yes

TypeSafeMatrix

a + b	$\begin{pmatrix}DX\\DY\end{pmatrix}$	$\begin{pmatrix}DY\\DZ\end{pmatrix}$	$\Delta\begin{pmatrix}DX\\DY\end{pmatrix}$	$\begin{pmatrix}VX\\VY\end{pmatrix}$
$(DX\ DY)^T$	no	no	yes	no
$(DY\ DZ)^T$		no	no	no
$\Delta(DX\,DY)^T$			yes	no
$(VX\ VY)^T$				yes

A * B	jac	jac^T	cov	vector
jac	maybe	no	maybe	maybe
jac^T	no	maybe	no	no
cov	no	maybe	no	no
vector	no	no	no	no



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Transformation and rotation matrices

How to define an isometry (a transformation consisting of rotation and translation):

```
tsm::Isometry<double, tsm::VehicleFrontAxleCoords, tsm::OdomCoords, 3> vehicle_T_odom{…};
```

Scalar type

Destination frame

Sourc fram

Dimension

Source frame

This enable … mpile … r correctne

```
// transf … vect … o vehicle c
vehicle_position = vehicle_T_odom * odom_position;
```

Valid or not?

```
vehicle_velocity = vehicle_T_odom * odom_velocity;
```

Velocities should not be translated => Compile error

```
vehicle_velocity = vehicle_T_odom.linear() * odom_velocity;
```

```
vehicle_delta_vector = vehicle_T_odom.linear() * odom_delta_vector;
```


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User-defined index structs

- Until now, we mainly used 2d- and 3d cartesian position and velocity vectors in different coordinate frames
- A TypeSafeMatrix is not restricted to this use-case, it can also be used to represent other quantities

```
struct SLOPE : tsm::NonCartesianIdxType<si::Metre> {};    // slope of a line fit
struct OFFSET : tsm::NonCartesianIdxType<si::Metre> {};  // offset of a line fit
```

- Operations that are only allowed for cartesian vectors:
  - Transformation to a different frame
  - norm(), cross(), dot()

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Auto variables and expression templates

```
// method returning by value
Eigen::Vector3d calculateOffset() {...}
Eigen::Vector3d vectorA = Eigen::Vector3d::Ones();
```

```
double result = (vectorA + calculateOffset()).norm();
```

```
auto sum = vectorA + calculateOffset();
double result = sum.norm();
```

```
const auto& sum = vectorA + calculateOffset();
double result = sum.norm();
```


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How to prevent usage of MatrixExpression

```
template<class T>
void staticAssertIfLvalueMatrixExpression() {
  static_assert(!IsTsmMatrixExpression<std::decay_t<T>> || std::is_rvalue_reference<T>::value);
}
template<class Leaf>
class MatrixBase {
  template<class Other>
  auto operator+(Other&& other) const& -> detail::MatrixExpression<…> {
      detail::staticAssertIfMatrixExpression<Leaf>();
      detail::staticAssertIfLvalueMatrixExpression<decltype(other)>();
      return {underlying() + std::forward<Other>(other).underlying()};
  }
  template<class Other>
  auto operator+(Other&& other) const&& -> detail::MatrixExpression<…> {
      detail::staticAssertIfLvalueMatrixExpression<decltype(other)>();
      return {std::move(*this).underlying() + std::forward<Other>(other).underlying()};
  }
};
```


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Supported operations

A.**uncheckedConstMatrix**()	const access to underlying matrix (Eigen / blaze)
A.**uncheckedMutableMatrix**()	mutable access to underlying matrix (Eigen / blaze)
A.**asDeltaVector**()	convert the vector to a delta vector
A.**uncheckedAsNonDeltaVector**()	convert the vector to a non-delta vector
A.**narrowing_cast**<Scalar>()	convert (lossy) the matrix to use another scalar type (e.g. int32_t → int16_t)
A.**widening_cast**<Scalar>()	convert (lossless) the matrix to use another scalar type (e.g. int16_t → int32_t)
A.**coeffSi**<ROW_IDX, COL_IDX>()	const access to individual matrix element as si unit
A.**coeffSiRef**<ROW_IDX, COL_IDX>()	non-const access to individual matrix element as si unit
A.**at**<ROW_IDX, COL_IDX>()	const access to individual matrix element as scalar
A.**at**<ROW_IDX, COL_IDX>()	non-const access to individual matrix element as scalar
A.**row**<ROW_IDX>()	const access to a row in a matrix
A.**assignRow**<ROW_IDX>(b)	assign a row in a matrix
A.**col**<COL_IDX>()	access a column in a matrix
A.**assignCol**<COL_IDX>(b)	assign a column in a matrix
A.**block**<RowTypeList, ColTypeList>()	const access to a block in a matrix
A.**assignBlock**<RowTypeList, ColTypeList>(B)	assign a block in a matrix

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Supported operations

A.**transpose**()	return the transpose of the matrix
A **+ -** B	Addition / substraction of 2 matrices (have to be compatible)
A ***** B	multiplication of 2 matrices
a.**dot**(b)	scalar product of 2 cartesian delta-vectors
A.**determinant**()	determinant of a matrix (only available for dim < 4)
a.**norm**()	calculates the L2-norm (length) of a cartesian delta-vector
a.**squaredNorm**()	calculates the square L2-norm of a cartesian delta-vector
a.**cross**(b)	calculates the cross product of 2 cartesian delta-vectors
a.**head**<n>()	returns the first n elements of the vector
A.**inverse**()	Calculates matrix inverse (only for dim <= 4)
a ***** si-unit; a **/** si-unit; a ***** scalar; a **/** scalar;	divides a cartesian delta-vector by a si-unit / scalar
A.**setRowTo**<ROW_IDX>(scalar)	set a row to a value
A.**setColTo**<COL_IDX>(scalar)	set a column to a value
A = MatrixType::**Ones**(); ::**Zero**(); **Identity**();	Matrix expression where each entry is 1 / 0 / Identity matrix
a = VectorType::**Unit**<ROW_IDX>();	Set vector to the unit vector where only ROW_IDX is 1
A.**setIdentity**(); A.**setOnes**(); A.**setZero**();	set the matrix to identity / 1 / 0

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What we get with TypeSafeMatrix

- Expressive (and enforced) names for vector / matrix entries
- Protection against out-of-bounds access (compile-time)
- Full support for physical units in vectors / matrices
- Compatibility check of index structs for all matrix operations (stronger condition than physical units-check)
- Include notion of coordinate frames
    - Possibility to specify a coordinate frame for a vector and source and dest coordinate frame for transformations
- Abstraction of underlying linalg library

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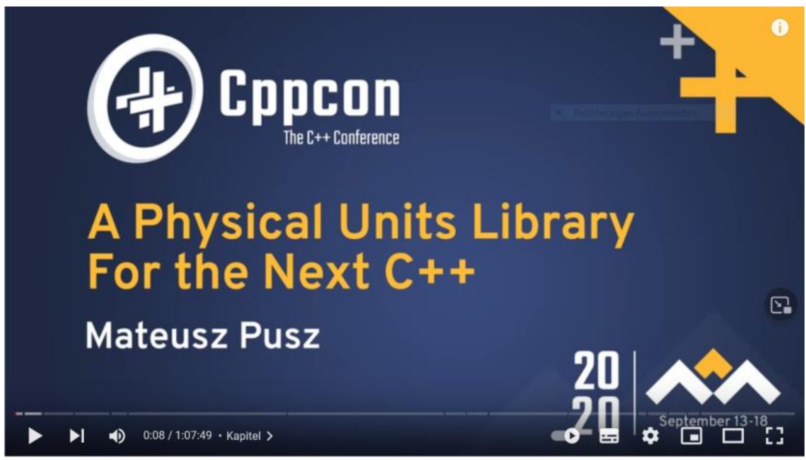
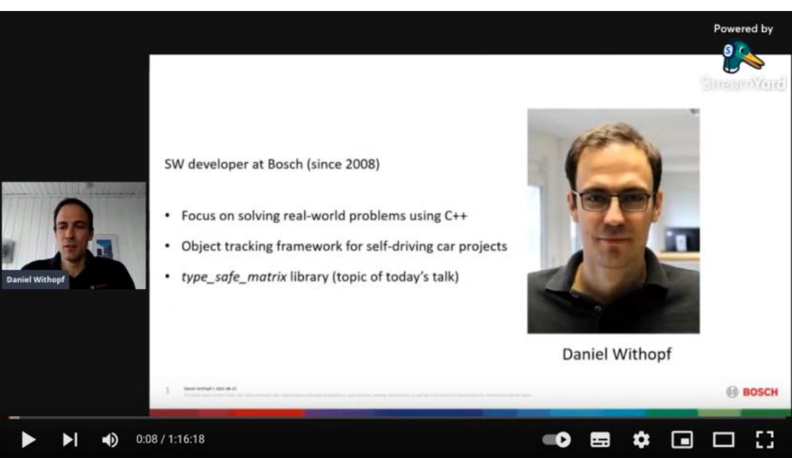
Physical units library http://wg21.link/P1385



https://www.youtube.com/watch?v=7dExYGSOJzo

Earlier version of my talk, contains more details about object tracking



https://www.youtube.com/watch?v=J6H9CwzynoQ

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Example how Uber / Aurora use their units library

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Thank you for listening, looking forward to your questions!

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