




Typing types at type level

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November 15, 2019

T

`typename T`

```
template<typename T>  
struct pi  
{ using type = typename T::first_type; };
```

```
template<typename T>
struct pi
{ using type = typename T::first_type; };

template<typename T>
using pi_t = typename pi<T>::type;
```

$expr \ni e ::= n \mid x \mid x := e \mid e = e \mid e + e \mid e - e$

$expr \ni e ::= n \mid x \mid x := e \mid e = e \mid e + e \mid e - e$

$n \in \mathbb{N}, x \in \mathbf{Vars}$

```
template<typename... Args>  
struct type {};
```

```
using int_t = type<int>;  
using bool_t = type<bool>;
```



```
template<typename... Args>
struct type {};

using fail_t = type<>;

using int_t = type<int>;
using bool_t = type<bool>;
```

```
template<typename... Args> struct expr {};
```

```
template<typename... Args> struct expr {};  
  
template<int n>  
using con = expr<std::integral_constant<int, n>>;
```

```
template<typename... Args> struct expr {};  
  
template<int n>  
using con = expr<std::integral_constant<int, n>>;  
  
template<typename e1, Opr op, typename e2>  
using binop = expr<std::integral_constant<Opr, op>,  
                  e1, e2>;
```

```
template<typename... Args> struct expr {};  
  
template<int n>  
using con = expr<std::integral_constant<int, n>>;  
  
template<typename e1, Opr op, typename e2>  
using binop = expr<std::integral_constant<Opr, op>,  
                  e1, e2>;  
  
template<int n>  
struct var_t { constexpr static int value = n; };  
  
template<int n>  
using var = expr<var_t<n>>;
```

```
template<typename... Args> struct expr {};  
  
template<int n>  
using con = expr<std::integral_constant<int, n>>;  
  
template<typename e1, Opr op, typename e2>  
using binop = expr<std::integral_constant<Opr, op>,  
                  e1, e2>;  
  
template<int n>  
struct var_t { constexpr static int value = n; };  
  
template<int n>  
using var = expr<var_t<n>>;  
  
template<int x, typename e2>  
using assign = expr<var<x>, e2>;
```

$$\frac{n \text{ is integer}}{n \rightarrow \text{integer}}$$

```
template<int n>  
constexpr auto elab(con<n>&&) -> int_t;
```



```
template<int n>
constexpr auto elab(con<n>&&) -> int_t;

static_assert(std::is_same_v<decltype(
    elab(std::declval<con<n>>())
), int_t>,
    "Constants are integers.");
```

$$\frac{x \in \Gamma}{\Gamma, x \rightarrow \Gamma, \Gamma(x)}$$

```
template<typename... Args>  
struct list {};
```

```
template<typename... Args>  
struct list {};
```

```
template<typename... Args0, typename... Args1>  
constexpr auto operator+(list<Args0...>&&  
                          list<Args1...>&&)  
    -> list<Args0..., Args1...>;
```

```
template<typename... Args>
struct list {};
```

```
template<typename... Args0, typename... Args1>
constexpr auto operator+(list<Args0...>&&,
                        list<Args1...>&&)
    -> list<Args0..., Args1...>;
```

```
template<typename T>
constexpr auto has(T&& t, list<>&&) -> std::false_type;
```

```
template<typename T, typename... Args>
constexpr auto has(T&& t, list<Args...>&&)
    -> std::bool_constant<
        (std::is_same_v<T, Args> || ...)
    >;
```

$\Gamma(x) = \text{integer} \quad :\Leftrightarrow \quad \text{list}\langle \text{std}::\text{pair}\langle \text{var}\langle 0 \rangle, \text{int_t} \rangle \rangle$

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```
template<int n>  
constexpr auto elab(con<n>&&) -> int_t
```

$\Gamma(x) = \text{integer} \quad :\Leftrightarrow \quad \text{list}\langle\text{std}::\text{pair}\langle\text{var}\langle 0 \rangle, \text{int_t} \rangle\rangle$

```
template<typename Env, int n>  
constexpr auto elab(Env&&, con<n>&&)  
    -> std::pair<int_t, Env>
```


$$\frac{x \in \Gamma}{\Gamma, x \rightarrow \Gamma, \Gamma(x)}$$

```
template<typename Env, int x>  
constexpr auto elab(Env&&, var<x>&&)  
->
```

```
var<x>,  
Env;
```

```
template<typename Env, int x>
constexpr auto elab(Env&&, var<x>&&)
-> decltype(
    find_pair_by_component<0>(
        std::declval<var<x>>(),
        std::declval<Env>()
    )
);
```

```
template<typename Env, int x>
constexpr auto elab(Env&&, var<x>&&)
-> detail::elab_var_t<decltype(
    has_nth_component<0>(
        std::declval<var<x>>(),
        std::declval<Env>()
    )
)>::value, Env, var<x>>;
```

```
template<bool c, typename T, typename D>  
struct elab_var;
```

```
template<bool c, typename T, typename D>  
using elab_var_t = typename elab_var<c, T, D>::type;
```

```

template<bool c, typename T, typename D>
struct elab_var;

template<bool c, typename T, typename D>
using elab_var_t = typename elab_var<c, T, D>::type;

template<int x, typename... Args>
struct elab_var<false, list<Args...>, var<x>>
{
    static_assert(x == 0 && false,
        "Does not support free variables.");

    using type = std::pair<fail_t, list<>>;
};

```

```
template<int x, typename... Args>
struct elab_var<true, list<Args...>, var<x>>
{
    using x_type = decltype(find_nth_component<0>(
        std::declval<var<x>>(),
        std::declval<list<Args...>>()));
};
```

```
template<int x, typename... Args>
struct elab_var<true, list<Args...>, var<x>>
{
    using x_type = decltype(find_nth_component<0>(
        std::declval<var<x>>(),
        std::declval<list<Args...>>()));

    using var_type = std::decay_t<decltype(
        std::get<1>(std::declval<x_type>()))>;

};
```



```

template<int x, typename... Args>
struct elab_var<true, list<Args...>, var<x>>
{
    using x_type = decltype(find_nth_component<0>(
        std::declval<var<x>>(),
        std::declval<list<Args...>>()));

    using var_type = std::decay_t<decltype(
        std::get<1>(std::declval<x_type>()))>;

    using type = std::conditional_t<
        !std::is_same_v<var_type, fail_t>,
        std::pair<var_type, list<Args...>>, fail_t>;
};

```

Hacking on it: <https://godbolt.org/z/vc88g4>

